COUNCIL OF EUROPE CONSEIL DE L'EUROPE

Language Policy Division
Division des Politiques linguistiques

Contents

Language in Mathematics?
A comparative study of four national curricula

Sigmund Ongstad (ed.), Brian Hudson (in cooperation with Peter Nyström), Birgit Pepin and Mihaela Singer

Languages across the curriculum within
Languages of Education

Language Policy Division, Strasbourg
www.coe.int/lang

Language in Mathematics?
A comparative study of four national curricula

Sigmund Ongstad (ed.), Oslo University, Norway
Brian Hudson (in co-operation with Peter Nyström), Umeå University, Sweden
Birgit Pepin, University of Manchester, United Kingdom
Mihaela Singer, Institute for Educational Sciences, Romania

Intergovernmental Conference

Languages of schooling within a European framework for Languages of Education: learning, teaching, assessment

Prague 8-10 November 2007

Organised by the
Language Policy Division, Council of Europe, Strasbourg
in co-operation with the
Ministry of Education, Youth and Sports, Czech Republic

Language Policy Division
DG IV – Directorate of School, Out-of-School and Higher Education

Contents

Language in Mathematics?

A comparative study of four national curricula

Sigmund Ongstad

Background

This short, somewhat summative study builds on four co-ordinated small-scale investigations of the explicit role of language and communication in mathematics curricula in England, Norway, Sweden and Romania. The study aims at addressing major, relevant key issues for an overall, international framework for language(s) of schooling. The background papers and texts hence consist of a study of each country's curriculum (Pepin, 2007a, Hudson and Nyström, 2007, Singer, 2007a and Ongstad, 2007a). The Swedish and the Romanian studies both have an attachment. In the former, particular tasks for evaluating language in mathematics are discussed. The latter gives a rather detailed overview over how language and communication is discursively positioned within the national curriculum in mathematics in Romania.

Finally there are two other texts published separately, one by B. Pepin that compares mathematics education in United Kingdom, Germany and France (Pepin, 2007b), and one by F. M. Singer that discusses the role of cognition in relation to language (Singer, 2007b). A longer paper by S. Ongstad, published separately, will sum up how language and communication is positioned within mathematics education on the curricular level in more general terms (Ongstad, 2007b). This overarching text will even suggest strategies for how LAC in mathematics can contribute to a general framework for language(s) of schooling. The paper will to some extent build on a work published in Educational Studies in Mathematics, *Mathematics and Mathematics Education as Triadic Communication?* (Ongstad, 2006).

England

Primary school in England ranges from age 5 to 11 (with focus on two so-called Key Stages, KS1 and KS2), whereas secondary (comprehensive) school ranges from age 11 to 16 (KS3 and KS4). Study programmes in the National Curriculum describe ‘what pupils should be taught’. What are called ‘attainment targets’ (AT) give expected standards of performance (as an outcome of teaching and learning). In mathematics there are four: using and applying mathematics; number and algebra; shape, space and measures; and handling data (HD). All the ATs consist of eight level descriptions of increasing difficulty, plus a description for exceptional performance above level 8.

There are four Key Stages for learning achievement along the years. KS3 and KS4 are described in The Secondary National Strategy. KS3, which seems closest to the end of compulsory schooling in other European countries, aims to raise standards by strengthening teaching and learning across the curriculum for all 11–14 year olds.

Except for formulations such as *mathematics provides opportunities for pupils to develop the key skills of: Communication, through learning to express ideas and methods precisely, unambiguously and concisely* and *Working with others, through group activity and discussions on mathematical ideas* language and communication is not really a significant issue in the national curriculum for mathematics in England at KS3. Still, one can find other formulations that reveal consciousness of the role of language and communication, such as *Interpret, discuss and synthesise geometrical* *information*, *communicate mathematically making use of geometrical diagrams and related explanatory text, Use precise language and exact methods to analyse geometrical configurations, and justify their choices.*

Pepin concludes that:

> (...) the National Curriculum (statutory) as well as the National Strategy (non-statutory), are concerned about language and communication for the teaching and learning of mathematics. Interestingly, the National Strategy seems to be mainly anxious about children learning the right kind of vocabulary in mathematics, such as for example *inverse, equivalence, equality, proportionality, congruence, similarity, linearity*, and so on.

There is a certain emphasis on communicational aspects such as discussion and interpretation, reasoning and proof (nevertheless still focusing language elements such as *if ...then, because, therefore, implies ...*, or *what if ...?* And *why?*). Although the NC claims that to communicate mathematically, including the use of precise mathematical language is at the heart of the endeavour, Pepin's impression is that overall it is the language aspect that is highlighted and emphasised, rather than communication. She also doubts whether curricular intentions are followed up in classrooms.

While curricular descriptions of both literacy, language and communication and of mathematics in *primary* education contains many ideas about relationships between language and mathematics, this connection seems to have been clearly downsized in secondary education/KS3. Here mathematics as such is the focus.

Sweden

In considering the aim of mathematics and its role in education, compulsory schools in Sweden have the task of:

> (...)providing pupils with the knowledge in mathematics needed for them to be able to make well-founded decisions when making different choices in everyday life, in order to be able to interpret and use the increasing flow of information and be able to follow and participate in decision-making processes in society. It is intended that the subject should provide a sound basis for studying other subjects, for further education and lifelong learning (Skolverket, 2007).

Hudson and Nyström (2007) find that mathematics as part of the wider culture and education is stressed in terms of giving an insight into the subject's historical development, its significance and role in society. Also:

The subject aims at developing the pupil's interest in mathematics, as well as creating opportunities for communicating in mathematical language and expressions. It should also give pupils the opportunity to discover aesthetic values in mathematical patterns, forms and relationships, as well as experience, satisfaction and joy in understanding and solving problems (Skolverket, 2007).

Although the importance to practise and communicate mathematically in meaningful and relevant situations are emphasised, the chapter concerning *goals to aim for* gives no explicit reference to communication. Hudson and Nyström find this rather surprising in view of the explicitly stated aims. Further, there are no explicit references to language and communication aspects in the section on the structure and nature of the subject or in the goals to be attained either by the end of the early phase or at the very end of compulsory school.

It should be noted though that there is explicit reference to the importance of *oral* communication in the criteria under the section on the student's ability to use, develop and express mathematical knowledge: *An important aspect of knowing mathematics is the student's ability to express her/his thoughts orally and in writing, with the help of the mathematical language of symbols and supported by concrete material and pictures.*

Swedish examples of mathematical *tasks* particularly suited for communication can be found in the national *assessment* system. Swedish national tests in mathematics are designed to cover a broad spectrum of the syllabus, they are fairly low-stakes and to a high degree aligned with the curriculum. Examples can be found in Appendix 1 (Hudson and Nyström, 2007).

Romania

In her study, Singer (2007a) found that new philosophies of education promoted by the National Curriculum put more emphasis on language across the curriculum. Thus, among the four framework objectives for mathematics in compulsory education, there is one devoted explicitly to communication: Knowledge and use of mathematical concepts, Development of exploration, investigation and problem-solving capacities, <u>Communicate using mathematical language</u> and Develop interest and motivation for studying mathematics and applying them to various contexts.

After investigating in detail the relationship between mathematics on the one hand and language and communication on the other hand in the different parts of the written curriculum, Singer concludes in six points. (These language and communication aspects are to some extent based on concepts developed in earlier texts by S. Ongstad and by H. Vollmer.):

> *Language as direct communication, as a way to exchange ideas and interact with others, to jointly construct meaning in pairs, in small groups or in the classroom as a whole, to negotiate meaning.* Among the four framework objectives, one is devoted to communicating using the mathematical language. The reference objectives for "Communicating using the mathematical language", as for the other framework objectives, are constructed in progression from grade one to the last grade of compulsory education.
>
> *Language as an expression of understanding and text comprehension*. Some reference objectives emphasise reading and writing mathematical texts, as well as decoding mathematics texts through the help of logical operators and quantifiers.
>
> *Language as (disciplinary) content (especially basic meanings/ terms and expressions).* Language as reflecting the structure of a topic or theme is emphasised within the framework objective "Knowledge and use of mathematical concepts", in the learning activities that are focused on terminology.
>
> *Language as discursive pragmatics, language as realisations of basic discourse functions (like naming, defining, describing, explaining, supporting, reporting, hypothesising, evaluating etc.)* is emphasised in various examples of learning activities that are provided within the subject curriculum for each grade.
>
> *Aspects related to language as creativity (the rheme/new part in theme-rheme or given-new dynamics) language as the tool and means for developing,*

creating and expressing new concepts and insights are targeted through the framework objective “Developing competencies in exploration, investigation and problem-solving”.

Language for reflecting (critically) on the subject and one’s own learning is emphasised through the framework objective “Developing interest and motivation for studying mathematics and applying it to various contexts”. The reference objectives focused on communication and on developing attitudes are fundamental for meta-cognition and aim at the very core of education in the 21st century (Singer 2007, this volume).

Norway

Ongstad (2007a) concludes that the new national curriculum for mathematics in Norway (LK06) in use from 2006 onwards, gives absolute priority to disciplinarity in the three parts 'objectives', 'subject areas' and 'competence aims'. In the fourth part, 'basic skills', language and communication, or 'discursivity', is by far the most significant pattern. *This leads to a schism that is the plan's most significant pattern* and creates uncertainty about the plan's main intention. Is it the objectives or is it the skills? The basic-skill chapter, where competences for language and communication dominate, can be read as a way of 'intruding' mathematics (and even the other school subjects in the curriculum), and force them to be *tools* for and mediate a wider enculturation (or perhaps even *'Bildung'*) rather than being an isolated, purely disciplinary knowledge. An overall conclusion can be that the aspects in question mainly are kept *separate*.

Ongstad's study even compares the two last national curricula. If there is any significant 'developmental line' from the former curriculum from 1997 (L97) to the current, LK06, what seems lost is the relative clear *conceptual* orientation found in L97. One implication is that the KL06 seems to put less weight on a conscious *semantic* based epistemology that could have opened a door between language and mathematics.

In general, a rather formal and quite general 'communicative' approach seems to have won a pyrrhic victory, and mainly in a particular part of the curriculum. This kind of schism is probably strongest in the Norwegian curriculum, but is nevertheless even an overall tendency in the four investigated curricula. However, if future Norwegian evaluations actually will end up focusing on overall basic skills (rather than purely mathematical ones), the communicational ambitions might become relevant further down the road.

A general interpretation

All four curricula have some, but not many explicit references to language and communication. These are mostly found in the general parts (introductions) where the discipline is related to learners, 'world' and society ('lifeworlds'). The closer one gets to specific goals and aims (often signalled as 'bullet points') the more weight is put on mathematics as such. While the English curriculum has quite strong references to language and learning across the curriculum for the primary level and appears to be quite balanced, the split and the imbalance between mathematics and communication are significant at what is called KS3 (in the lower part of secondary education). The same tendency to keep mathematics and language separate is found in the Swedish and the Norwegian curricula. The Romanian curriculum in mathematics seems, at least rhetorically, willing to pay more attention to communication as part of mathematics as a didactic enterprise.

In Norway, the idea of communication in mathematics rather stems from general curricular and didactic tendencies *outside* the discipline, an idea that is generally *imposed* on the different school subjects. This leads to a significant *schism* in the text in that each part of the curriculum has different intentions and implications. The English curriculum in mathematics could be said to stress *language* more than communication, a tendency for instance visible in prioritising clarity and preciseness when working with key concepts. The Swedish curriculum is similar to the Norwegian in that the introduction gives room to language and communication, but more or less skips this interest in the other parts. A big difference, though, is that the Norwegian curriculum has explicitly *altered* the relationship between the discipline and more overall curricular aims (that is, *basic skills*). In this particular part of the curriculum, mathematics is forced to see itself as a means, an ambition that is hardly supported in the other, more 'crucial' parts.

All the curricula have very brief descriptions of 'disciplinarity', mostly given as short, concrete cues or bullet points. This 'discourse' is dominated by disciplinary 'nouns', concepts that most teachers are familiar with. The nouns in these points function implicitly as part of speech acts that seem to give clear directives for what the syllabus is supposed to be. By the same token, the curricula are rather unclear about what is important, as there is hardly any hierarchical order between the many aspects within this part of the texts. Although all national curricula in the general parts clearly give priority to the needs and the interests of the learner and society, the discipline of mathematics seems able to establish its *own* agenda within this 'horizontally' structured, curricular framework. Clear ambitions of introducing language and communication in curricula for mathematics are, therefore, generally not really backed up.

An overall strategic discussion of what is at stake will, as mentioned, be given in Ongstad (2007b), which again will be part of a general analysis of language across the curriculum. As a start the following challenges should be considered.

Some key issues

A All the inspected curricula give mathematics a key role in culture and society, and stress (to different degrees) important connections between mathematics as a discipline and a school subject on the one hand, and language and communication on the other hand. Whether this is really 'meant' is hard to say, since there are symptoms that just pay lip service to the importance of communication. On the other hand, the rhetorical phrasing of this aspect could be related to more hidden power games and tugs of war in reform processes.

B. In all the four countries basic skills or key competencies have been given priority in the overall general, national curriculum. Only Norway has followed up this intention in the school subject sections by giving the basic skills (rather formally) a key role in all the curricula and hence also in mathematics. This raises the problem of priority (signalled by hierarchical structure) between different curricular parts. With the worldwide new 'design' of curricula ('shortness' as 'clearness') and with ongoing silent battles between summative and formative evaluation, it is not obvious for teachers how they should prioritise, or to what extent they should try to integrate the two ambitions.

C. There are reasons for believing that mathematics has more problems than most other school subjects with the integration or the interface between mathematics and language and communication. There is, for instance, a strong will in the disciplinary parts of the mathematics curricula to describe mathematics rather than relating mathematics to different communicative contexts. Further, perceptions of what language and communication might be seem, if not outdated, at least rather fragmented and coincidental. Finally, it is tempting to believe that among mathematics teachers in secondary education and among others that contribute to designing a curriculum, there exists a strong priority of *disciplinarity*. The profile of the elements of the school subject is often conceived as a 'skyscraper' rather than as a row of ‘terraced houses'. Within such a 'vertical' paradigm there is probably less room for seeing mathematic education as a compound of elements of aspects from other fields of knowledge (such as linguistics and sociology). If this is the case, it is quite thought-provoking that among many *theorists* in mathematics education language, communication and semiotics, in short, discursivity, is seen as crucial or even inevitable.

Which aspects are important?

There are several important aspects of language and communication that deserve further investigation and clarification, such as *concept, context, discursivity* and *semiotics*. However, these cannot be described in theoretical isolation. They need to be related to other major, didactic, contextual processes, such as *teaching*, *disciplining* and *learning*.

A problem with educational understandings of mathematical *concepts* is that a more cognitive and semantic view has dominated among educators, who are mostly recruited from the disciplinary side. Even if socio-constructivism seems relatively strong in mathematics in many European teacher educations, this direction seems to have had less impact on how the different curricula have phrased the core concepts that are important for integrating language and communication elements in mathematics as a school subject.

Partly related to this problem is the role of *context*. While the sociological awareness of the importance of social background regarding children's conceptualising was increasing in the 1970s, the interest in developing a more advanced understanding seems to have evaporated during the 1980s and 1990s. For the writing of curricula and textbooks this loss in insight has lead to a lack of interest in the cultural significance of key concepts in the process of learning.

However, context is important in a much wider sense as well. A recent evaluation among teachers of mathematics in the UK made it clear that teachers' awareness of the role of context is increasing. A problem is that with a traditional, simplistic understanding of context as some kind of box, as a 'place' or physical environment, a discursive, semiotic and communicational perspective seems harder to grasp. In such a view a mathematical concept can often 'belong' to a particular mathematical genre or discourse (which hence becomes a context). Further this communicational form may belong in a particular, educative discursive setting. And the school as institution will be part of a certain kind of society, and so on. This never-ending *embeddedness* can hardly be handled in a didactic adequate way without a broader understanding of context as discursive, semiotic and communicational.

Learning mathematics can thus be seen to be permeated by constant *discursivity*: one cannot not use mathematical genres, one cannot not communicate mathematically, any mathematical act is in some sense an utterance, any concept is both mathematical *and* linguistic, mathematics is not just a language in its own, it is even a 'languaging', discursive activity, and it contributes to 'language'.

A position that can help bridge the gaps between language and mathematics is *semiotics*. In this perspective learning mathematics can be seen as a cultural sign-process. The pre-school child has been enculturated through an ethno-mathematical experience where letters, words, figures, pictures, in short *signs*, are the basic entities. The semiotic unit sign may constitute a simple starting point for any mathematics in schools, irrespective of whether a national written curriculum takes this fact into consideration or not. The wider scope of 'semiotics' may also help to reduce a somewhat negative attitude to 'language' in some disciplines. Still, *semiotics* is a fairly unknown word and one cannot expect it to be commonly used and understood. Its main function could therefore be to support the idea that it is necessary to *extend* the horizon from 'just' verbal language to semiotic *communication*.

If the above four focused concepts are presented in isolation, they will not be of much help in moving mathematics and communication closer together. *Teaching* mathematics needs to be seen as communication. *Developing the discipline/school subject* of mathematics ('disciplining') needs to be seen as communication, and *learning* mathematics needs to be seen as communication. However, expressing these three perspectives separately does not make them *different* kinds or separate processes of communication. Although it is possible (and even sensible) sometimes to focus on them separately, they happen *simultaneously*. For this to be better understood, doing mathematics, whether as research or in learning, should be seen *as* communication, not just as something that to some extent can be mediated by language. To make this clear for teachers of mathematics (and others), the *implicit* language parts of curricula, textbooks, teaching and learning have to be "lifted to the surface" by a conceptual framework

Ongoing research in mathematics, and everyday use of mathematics in education and in society as a whole, contribute semiotically and communicatively to culture in an integrative way, weaving together disciplinary and discursive aspects. *Which* aspects will be most important, relevant and adequate in different contexts, is a crucial *didactic* question. However, that can hardly be convincingly addressed without a more subtle conceptual framework.

Just as it has been a problem for pedagogy as an educational discipline in teacher education to be both specific and general when handling different subject content in education, it will be a challenge for the language of schooling to find a reasonable conceptual balance between specific mathematicallity and general disciplinarity. A key question will be which concepts are adequate for describing overall communicational patterns within school subjects, and which are just valid and relevant in certain sub-fields. That should be answered by relating explicitly to disciplinarity while investigating communicational conditions for school subjects across borders in Europe.

References

Hudson, B. and Nyström, P. (2007) *Language across the mathematics curriculum in Sweden: (*in this volume): Council of Europe, Strasbourg: www.coe.int/lang

Ongstad, S. (2006) Mathematics and Mathematics Education - Language and/or Communication? Triadic Semiotics Exemplified. *Educational Studies in Mathematics, 61/1-2*.

Ongstad, S. (2007a) *Language and communication in Norwegian curricula for mathematics. (*in this volume): Council of Europe, Strasbourg : www.coe.int/lang

Ongstad, S. (2007b) *Disciplinarity versus discursivity*? Council of Europe, Strasbourg : www.coe.int/lang

Pepin, B. (2007a) *Language across the mathematics curriculum in England. (*in this volume): Council of Europe, Strasbourg : www.coe.int/lang

Pepin, B. (2007b) *Culture, language and mathematics education: aspects of language in English, French and German mathematics education. (*in this volume): Council of Europe, Strasbourg : www.coe.int/lang

Skolverket (2007) *KUSINFO 2006/07*, http://www3.skolverket.se/ki03/front.aspx?sprak=EN

Skolverket (2005) *National Assessment and Grading in the Swedish School System*, http://www.skolverket.se

Singer, M. (2007a) *Language across the mathematics curriculum in Romania*. *(*in this volume): Council of Europe, Strasbourg : www.coe.int/lang

Singer, M. (2007b) *Language across the mathematics curriculum: some aspects related to cognition. (*in this volume): Council of Europe, Strasbourg : www.coe.int/lang

Utdanningsdirektoratet (2007) *Mathematics Subject Curriculum.* (LK06). Visited 31.05.07.
www.udir.no/.../Fastsatte_lareplaner_for_Kunnskapsloeftet/english/Mathematics_subject_curriculum.rtf

Language across the mathematics curriculum in England

Birgit Pepin

> "Mathematics education begins in language, it advances and stumbles because of language, and its outcomes are often assessed in language." (Durkin, 1991, p.3)

Language is important for learning mathematics with understanding, and it is the major tool mediating interactions between learners, and between teachers and learners. In some countries, it is argued (Setati, 2005- South Africa) that language use is more a function of politics than it is of cognition and communication. Setati points to the significance of language as power in mathematics-education settings, and thus to the need for further research into the relationship between language and the teaching and learning of mathematics, 'to embrace the political dimensions of this relationship' (p.74, ibid). Whilst acknowledging his concerns, but without going into the political debate, I would like to address the *language* and *communication* aspects in the Mathematics National Curriculum for England and in the Key Stage 3 National Strategy documents. If we believe what Durkin says, there is no learning of mathematics without language and/or communication.

Before analysing the Mathematics National Curriculum for England in greater detail with respect to language and communication, I would like to outline its structure and contents. For the government, the National Curriculum 'lies at the heart' of their policies to raise standards and it is regarded as the 'statutory entitlement to learning for all pupils' (p.3). In England, primary school starts at age five and goes up to age 11 (Key Stages 1 and 2), whereas secondary (comprehensive) school ranges from age 11 to age 16 (Key Stages 3 and 4). Teachers are educated and trained according to these two age ranges. The structure of the National Curriculum for England is such that the programmes of study set out 'what pupils should be taught', and the attainment targets set out 'the expected standards of pupils' performance' (p.6). In mathematics there are four attainment targets (AT):

- Using and applying mathematics (UAM);
- Number and algebra (NA);
- Shape, space and measures (SSM);
- Handling data (HD).

Attainment targets consist of eight level descriptions of increasing difficulty, plus a description for exceptional performance above level 8. Each level description describes "the types and range of performance that pupils working at that level should characteristically demonstrate" (DfEE, 1999, p.7)

The following table gives an outline of what students should have attained at which age:

	Range of levels within which the great majority of pupils are expected to work		Expected attainment for the majority of pupils at the end of the Key Stage
Key Stage 1	1-3	At age 7	2
Key Stage 2	2-5	At age 11	4
Key Stage 3	3-7	At age 14	5/6
Key Stage 4		At age 16	National qualifications (GCSE)

However, there is also a part where 'learning across the curriculum' in a number of areas is outlined and 'the ways in which the teaching of mathematics can contribute to learning across the curriculum' (p.8).

The National Curriculum for England in Mathematics (DfEE, 1999) intends to promote 'key skills through mathematics' (p.8), in terms of learning across the curriculum, and the following relate to communication and language:

"… mathematics provides opportunities for pupils to develop the key skills of:

- *Communication*, through learning to express ideas and methods precisely, unambiguously and concisely
- …
- *Working with others*, through group activity and discussions on mathematical ideas
- … " (ibid, p.8)

It is not clear at this stage what it means to 'express ideas and methods precisely, unambiguously and concisely' and what kinds of opportunities learners have to learn how to do this, but communication and discussion are nevertheless mentioned as an important part of their learning of mathematics.

Learning to communicate mathematically is now generally seen by researchers (Pimm, 1987; Adler, 2001; Sfard, Nesher, Streefland, Cobb and Mason, 1998) and curriculum designers (NCTM, 1991, 2000; DfEE, 1999) as a central element of what it means to learn mathematics. Going through the programmes of study for the different Key Stages, the following mention is made with respect to language and communication:

Mathematics- The National Curriculum for England (www.nc.uk.net)

	General mention	Communicating	Reasoning/ elsewhere
Key Stage 1	“talking about…” “describing …” “using everyday words to describe position”		
Ma2 Number Ma3 (SSM)		“Use correct language, symbols and vocabulary associated with number and data” “Communicate in spoken, pictorial and written form, at first using informal language and recording, then mathematical language and symbols”	Explain … Describe … Discuss …
Key Stage 2			
Ma 2 (NA)		“Communicate mathematically, including the use of precise mathematical language” “… and describe …” “… explaining their method …”	“written methods”
Ma3 (SSM)			“use mathematical reasoning to explain features of shape and space” “visualise and describe 2-D and 3-D shapes …”
Key Stage 3			
Ma2 (NA)		“represent problems and solutions in algebraic or graphical forms …” “… justify … present …”	(written methods) “solve word problems about ratio and proportion …” “link a graphical representation of an equation to its algebraic solutions …”
Ma3 (SSM)		“Interpret, discuss and synthesise geometrical information …”	“explain and justify inferences …”

		"communicate mathematically making use of geometrical diagrams and related explanatory text" "Use precise language an exact methods to analyse geometrical configurations" "… justify their choices …"	
Ma4 (HD)		"interpret, discuss and synthesise …" "communicate mathematically …" "examine critically and justify …"	
Key Stage 4			
Ma 2 (NA)		"interpret and discuss …" "use notation and symbols correctly …"	(use of symbols) (index notation)
Ma 3 (SSM)		"interpret, discuss and synthesise …" (as above)	
Inclusion	(developing understanding) "using word descriptions and other stimuli to make up for …" (developing spoken and written English) "… effective opportunities for talk and that talk is used to support writing …." (ensuring access) "using accessible texts and materials …"		
Use of language across the curriculum	• Writing • Speaking • Listening • Reading "…pupils should be taught the technical and specialist vocabulary …"		
KS3 National Strategy: Framework for teaching mathematics: Years 7, 8 and 9	Checklist of important vocabulary "pupils are expected to use correct mathematical terms and notation and to talk about their insights rather than give single-word answers"		Reasoning skills- "these enable pupils to give reason for opinions and actions …and use precise language to reason."

	The NC statement on language suggests three areas to include in all subject teaching: • General accuracy in using language (spoken, written, read) • Technical terms and concepts appropriate to the subject; • Awareness of patterns of language. (also mention of language for pupils with communication difficulties)		

From the above there is evidence that both the National Curriculum (statutory) and the National Strategy (non-statutory) are concerned about language and communication for the teaching and learning of mathematics. Interestingly, the National Strategy seems to be mainly anxious about children learning the right kind of vocabulary in mathematics, such as for example *inverse, equivalence, equality, proportionality, congruence, similarity, linearity*, and so on.

Whilst the National Curriculum also mentions the mathematical vocabulary in order for pupils to be able to express themselves clearly, it nevertheless puts more emphasis on communicational aspects such as discussion and interpretation, and quite importantly reasoning and proof. Discussion is seen as one of the key skills when *working with others*, through group activities, for example. Where these discussions are to take place is not mentioned, because the teaching guidelines also emphasise skill training. However, *communicating mathematically, including the use of precise mathematical language* is at the heart of the endeavour, according to the NC, throughout the Key Stages. In terms of reasoning and proof, pupils' attention is drawn to statements involved in mathematical reasoning and proof, such as *if …then, because, therefore, implies …*, or *what if …?* And *why?* This is gradually increased in emphasis through the Key Stages.

My impression is that overall, it is the language aspect, rather than the communication aspect, that is highlighted and emphasised. However, in schools I see few teachers emphasising mathematical language, except in top sets. Mostly, rather colloquial expressions are used, such as 'top number' and 'bottom number' for numerator and denominator respectively. Furthermore, whilst lip service is paid to group work and working and communicating with others (in the National documents), in reality and in the classroom teachers spend very little time on developing such skills. In schools, the emphasis is on covering the curriculum and dealing with behaviour management issues (i.e. disruptive pupils). Getting pupils successfully through examinations is a high priority, and more communicational practices, which might involve group work, for example, or generally allowing children to discuss their work with one another, are seen as slowing down the process. In addition, and if pupils are not used to such pedagogic practice, teachers fear that pupils do not stay on task and do not sufficiently engage when they are allowed to discuss the work.

"Maths is the truly global language. With it, we convey ideas to each other that words can't handle – and bypass our spoken Tower of Babel." (Professor Alison Wolf, Head of Mathematical Sciences Group, Institute of Education, University of London) (DfEE, 1999, p.15)

References

Adler, J. (2001) *Teaching mathematics in multi lingual classrooms* , Dordrecht: Kluwer.

Department for Education and Employment (DfEE) (1999) *Mathematics- The National Curriculum for England*, London: DfEE and QCA.- www.nc.uk.net

DfEE (2001) *Key Stage 3 National Strategy*- Framework for teaching mathematics: Years 7, 8 and 9. London: DfEE.

Durkin, K. (1991) Language in mathematics education: an introduction. In Durkin, K. & Shire, B. (eds) *Language in mathematics education*, Milton Keynes: Open University Press.

National Council of Teachers of Mathematics (NCTM) (1991) *Professional Standards for teaching mathematics,* Reston, VA: NCTM.

NCTM (2000) *Principles and standards for school mathematics,* Reston, VA: NCTM.

Pimm, D. (1987) *Speaking mathematically: communication in mathematics classrooms,* London and New York: Routledge & Kegan Paul.

Setati, M. (2005) Mathematics education and language: policy, research and practice in multilingual South Africa, in Vithal, R., Adler, J. & Keitel, C. (eds) *Researching mathematics education in South Africa*, Cape Town, SA: HSRC.

Sfard, A., Nesher, P., Streefland, P., Cobb, P. & Mason, J. (1998) Learning mathematics through conversation: is it good as they say? *For the learning of mathematics*. London: Ward Lock Educational.

Language across the mathematics curriculum in Sweden

Brian Hudson and Peter Nyström

1. Steering System

The Swedish school system as described by Skolverket (2007) is a goal-based system founded on a high degree of local responsibility. The main responsibility lies with the municipalities and authorities responsible for independent schools.

The overall national goals are set out by Swedish Parliament and Government in:

- The Education Act
- Curricula
- Course syllabi for compulsory school etc.
- Program goals for upper secondary school

The National Agency for Education draws up and takes decisions on:

- Course syllabi for upper secondary school etc.
- Grading criteria for all types of Swedish school
- General recommendations

Every municipality develops a local school plan showing how the schools in that municipality are to be organised and developed. The curriculum, syllabi and school plan then allow the principals, teachers and students of individual schools the flexibility to adapt content, organisation and work methods to local conditions. The planning of these elements are laid out in the school's work plan. This work is followed up in annual Quality reports.

2. Curricula

The curricula for the compulsory school system are founded on a statement of fundamental values and tasks of the school related to the democratic basis of the society. The curricula are organised into three main phases based on the following Lärorplaner:

- Lpfö98 Curriculum for the Preschool
- Lpo94 Curriculum for the Compulsory School System, the Preschool Class and the Leisure-time centre
- Lpf94 Curriculum for the Non-compulsory School System

3. Goals

The knowledge to be acquired and developed by pupils is expressed in terms of goals to aim for and goals to attain. *Goals to aim for* express the direction the subject should take in terms of developing pupils' knowledge. They clarify the quality of knowledge which is essential in the subject. These goals are the main basis for planning teaching and do not set any limits to the pupils' acquisition of knowledge. *Goals to attain* define the minimum knowledge to be attained by all pupils in the fifth and ninth year of school. The goals thus set out a basic level of knowledge required in the subject from both these time perspectives.

Whilst national tests take place at grades 5 and 9 (age 16) of the Compulsory School phase, plans are currently at an advanced stage for national tests to be introduced at grade 3 also.

4. Grading system

The grades given in Compulsory school are:

- Pass with Special Distinction MVG
- Pass with Distinction VG
- Pass G

Goals to attain in the ninth year of school are used as the basis for assessing whether a pupil should be awarded the "Pass" grade. The majority of pupils will advance further and are expected to advance further in their learning also.

5. Mathematics Curriculum for Compulsory Schooling: what kind of explicit and implicitlanguage and learning expectations are there?

5.1 Aim of the subject and its role in education

In considering the aim of mathematics and its role in education, compulsory schooling has the task of:

…providing pupils with the knowledge in mathematics needed for them to be able to make well-founded decisions when making different choices in everyday life, in order to be able to interpret and use the increasing flow of information and be able to follow and participate in decision-making processes in society. It is intended that the subject should provide a sound basis for studying other subjects, for further education and lifelong learning.

The importance of mathematics as part of the wider culture and education is stressed in terms of giving an insight into the subject’s historical development, its significance and role in society. A central aim of the subject is seen in terms of developing the pupil’s interest in mathematics and aspects of language and communication are highlighted:

The subject aims at developing the pupil's interest in mathematics, as well as creating opportunities for communicating in mathematical language and expressions. It should also give pupils the opportunity to discover aesthetic values in mathematical patterns, forms and relationships, as well as experience satisfaction and joy in understanding and solving problems.

Opportunities for pupils to practise and communicate mathematically in meaningful and relevant situations are also emphasised: *The subject should give pupils the opportunity to practise and communicate mathematically in meaningful and relevant situations through actively and openly searching for understanding, new insights and solutions to different problems*.

5.2 Goals to aim for

However, in analysing the *goals to aim for* there is no explicit reference to communication to be found. This is rather surprising in view of the explicitly stated aims. However the teaching of mathematics should aim to ensure that pupils:

appreciate the value of and use mathematical forms of expression. Whilst "forms" of expression might include language such an interpretation is not necessarily inevitable.

Explicit reference to oral communication is, however, to be found in the following goal to aim for which refers to oral explanation and presentation of arguments for thinking, so that pupils should: *develop their ability to understand, carry out and use logical reasoning, draw conclusions and generalise, as well as orally and in writing explain and provide the arguments for their thinking.*

The remaining goals to aim for stress skills and conceptual understanding and there is no further explicit reference to language or communication.

5.3 Structure and nature of the subject

In a discrete section on the structure and nature of the subject there is an emphasis on mathematics as a living human construction:

"Mathematics is a living human construction involving creativity, research activities and intuition. Mathematics is also one of our oldest sciences and has been considerably stimulated by the natural sciences. The subject of Mathematics is based on the concept of number and space and studies concepts with well-defined properties. All mathematics contains some degree of abstraction. Similarities between different phenomena are observed and these are described in mathematical terms. A natural number is one such abstraction."

Further aspects to be emphasised include applications of mathematics to problems of everyday life, problem solving in general and cross-curricular relevance. However there are no explicit references to language and communication aspects. Interestingly this is in contrast to the parallel section in the curriculum for upper-secondary school which includes several such references.

5.4 Goals that pupils should have attained by the end of the fifth year in school (age 11)

The earliest phase for which expectations are explicitly stated in mathematics in Sweden is currently at the end of grade 5 though plans are at an advanced stage to introduce national tests at grade 3 (age 9).

Reference is made to the need for pupils to "describe" situations in relation to goals to be attained, although it is not made explicit what form such description might take, e.g. oral or in writing, etc. Neither is it clear whether such "description" refers to modelling, mathematising or communicating in general: *Pupils should have acquired the basic knowledge in mathematics needed to be able to describe and manage situations, and also solve concrete problems in their immediate environment.*

Within this framework, emphasis is placed on conceptual understanding, skills and methods, though there is one reference to "describing" important properties of geometrical objects.The list is not extensive and is listed in full below, in terms of "students should":

- have a basic understanding of numbers, covering natural numbers and simple numbers in fractions and decimal form,
- understand and be able to use addition, subtraction, multiplication and division, as well as be able to discover numerical patterns and determine unknown numbers in simple formulae,

- be able to calculate in natural numbers – in their head, and by using written calculation methods and pocket calculators,
- have a basic spatial understanding and be able to recognise and describe some of the important properties of geometrical figures and shapes,
- be able to compare, estimate and measure length, area, volume, angles, quantities and time, as well as be able to use drawings and maps,
- be able to read off and interpret data in tables and diagrams, as well as be able to use some elementary co-ordinates.

There is no explicit reference to language or communication in these goals to be attained by the end of this early phase of compulsory school.

5.5 Goals that pupils should have attained by the end of the ninth year in school (age 16)

Reference is made to the need for pupils to "describe" situations in relation to goals to be attained, although it is not made explicit what form such description might take, e.g. oral, in writing, etc: *Pupils should have acquired the knowledge in mathematics needed to be able to describe and manage situations, as well as solve problems that occur regularly in the home and society, which is needed as a foundation for further education.*

Within this framework, emphasis is placed on conceptual understanding, skills and methods, though there is one reference to "describing" important properties of geometrical objects. The list is not extensive and is listed in full below, in terms of "students should":

- have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form,
- have good skills in and be able to make estimates and calculations of natural numbers, numbers in decimal form, as well as percentages and proportions in their head, with the help of written calculation methods and technical aids,
- be able to use methods, measuring systems and instruments to compare, estimate and determine length, area, volume, angles, quantities, points in time and time differences,
- be able to reproduce and describe important properties of some common geometrical objects, as well as be able to interpret and use drawings and maps,
- be able to interpret, compile, analyse, and evaluate data in tables and diagrams,
- be able to use the concept of probability in simple random situations,
- be able to interpret and use simple formulae, solve simple equations, as well as be able to interpret and use graphs for functions describing real relationships and events.

There is no explicit reference to language or communication in these goals to be attained by the end of compulsory schooling.

6. Further issues to arise in the process of developing this paper

The strong influence of a hermeneutic tradition in Sweden needs to be stressed, i.e. the local plans of the municipilaties and schools represent interpretations of the national steering documents. Therefore, in order to get a more detailed picture of practice it would be necessary to make a closer study of course plans at the local municipal and school level, in addition to some in depth studies of a sample of schools.

During the process of developing this paper it became clear that assessment criteria are completely missing from the English-language versions of the curricula documents on the web. This is especially significant as there is explict reference to the importance of oral communication in the criteria under the section on the student’s ability to use, develop and express mathematical knowledge: *An important aspect of knowing mathematics is the student’s ability to express her/his thoughts orally and in writing, with the help of the mathematical language of symbols and supported by concrete material and pictures.*

The full assessment criteria are included in the next section.

6.1 Assessment in mathematics

6.1.1 Direction of assessment

The assessment of the student’s mathematical ability in mathematics concerns the following qualities: *The ability to use, develop, and express mathematical knowledge*. The assessment concerns the student’s ability to use and develop her/his mathematical “knowing” to interpret and deal with different kind of tasks and situations that can be found in school and society, for example the ability to discover patterns and relationships, suggest solutions, make (rough) estimations, reflect on and interpret her/his results and assess their reasonableness. Independence and creativity are important aspects to consider in assessment as well as clarity, thoroughness, and skill. An important aspect of knowing mathematics is the student’s ability to express her/his thoughts orally and in writing, with the help of the mathematical language of symbols and supported by concrete material and pictures.

The ability to follow, understand and scrutinise reasoning in mathematics. The assessment concerns the student’s ability to be receptive of (“ta del av”) and use information in oral as well as written form, for example the ability to listen to, follow and scrutinise explanations and arguments given by others. Furthermore, attention is given to the student’s ability to independently and critically decide on mathematically founded descriptions and solutions to problems found in different contexts in school and society.

The ability to reflect on the significance of mathematics in culture and society. The assessment concerns the student’s awareness (insight) and feeling for the value and limitations of mathematics as a tool and aid of assistance in other school-subjects, in everyday life and society and in communication between people. It also concerns the student’s knowledge about the significance of mathematics in a historical perspective.

6.1.2 Criteria (standards) for Pass with distinction

The student uses mathematical concepts and methods in order to formulate and solve problems. ('Formulate' can be interpreted with the help of aims to strive for, indicating that 'formulate' means the same as 'mathematise' or perhaps 'model'.)

- The student follows and understands mathematical reasoning
- The student makes mathematical interpretations of every day events or situations and conducts and gives a record using logical reasoning in the work, orally as well as in writing
- The student uses words, pictures and mathematical conventions in such a way that it is possible to follow, understand and scrutinise the thoughts being expressed
- The student demonstrates consistency ("säkerhet") in solving problems and uses different methods and procedures.
- The student can separate guesses and assumptions from what we know or are able to verify
- The student exemplifies, within a few areas, how mathematics has developed and been used through history and the significance it has in our time.

6.1.3 Criteria (standards) for Pass with special distinction

- The student formulates and solves different kinds of problems, and compares and evaluates the advantages and disadvantages of different methods
- The student shows "säkerhet" (security/confidence) in calculations and solving problems, and chooses and adjusts calculation methods and tools to the problem at hand
- The student develops problems and uses general strategies in planning and performing solutions, and analyses and reports in a structured way using a correct mathematical language
- The student listens to arguments presented by others and presents based on these arguments their own mathematically founded ideas
- The student reflects on the significance of mathematics in culture and social life

7. Tasks

Swedish examples of mathematical tasks particularly suited for communication can be found in the national assessment system. Swedish national tests in mathematics are designed to cover a broad spectrum of the syllabus, they are fairly low-stakes and to a high degree aligned with the curriculum. Examples can be found in Appendix 1.

8. Concluding comments

According to Skolverket (2005), political party views on national tests and grading in Sweden differ, with the conservative parties generally advocating awarding grades at an earlier age, more refined grading scales (more grade levels) and more national tests. In contrast, the left wing parties are averse to increased grading and national tests. This is seen as a difference in philosophy between advocates of assessment *of* learning in contrast to assessment *for* learning. Given a recent change of government in Sweden, the prediction by Skolverket (2005) of an increase in assessment *of* learning is taking place.

This small-scale analysis of the curriculum suggests that there could be significant potential for developing the role of language across the curriculum in mathematics in compulsory schooling. This could have significant implications for levels of student achievement and, if such development were to be implemented, would have significant implications for curriculum design, initial teacher education, continuing professional development and for priorities of policy makers in terms of the allocation of resources and in terms of value for money at the end of the day. At the very least, this situation seems to be worthy of a more detailed and in-depth study in the near future.

References

Skolverket (2007) KUSINFO 2006/07, http://www3.skolverket.se/ki03/front.aspx?sprak=EN

Skolverket (2005) National Assessment and Grading in the Swedish School System http://www.skolverket.se

APPENDIX 1

Tasks intended for oral and written communication in mathematics

Swedish examples of mathematical tasks particularly suited for communication can be found in the national assessment system. Swedish national tests in mathematics are designed to cover a broad spectrum of the syllabus. They are fairly low-stakes and to a high degree aligned with the curriculum.

Oral tasks

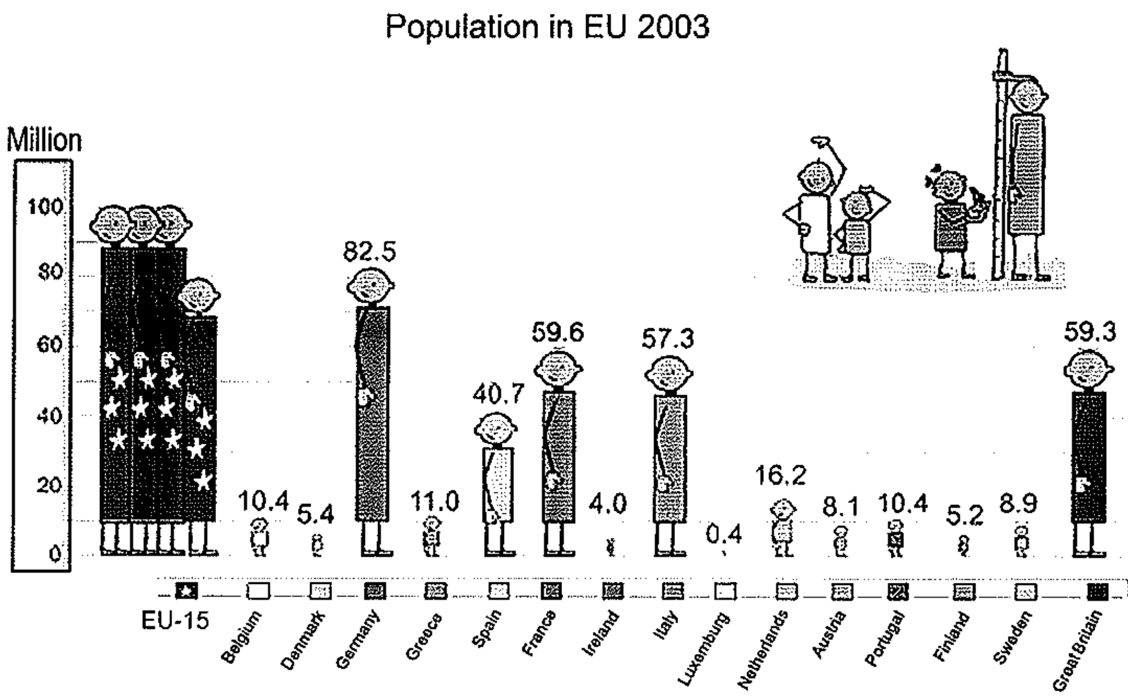
Two tasks or models from the national assessment system

Oral tasks in mathematics are exemplified by two kinds of models used in the Swedish national assessment system.

5, 6, 8	Calculating proportions (fractions or percent).
7, 9	Understanding what constitutes the whole, when making comparisons.
7, 10, 11, 12, c, d	Understanding the relationship population – population density – area.
a, b, e	Careful scrutiny of diagrams.
b, e	Understanding length scale, area scale.

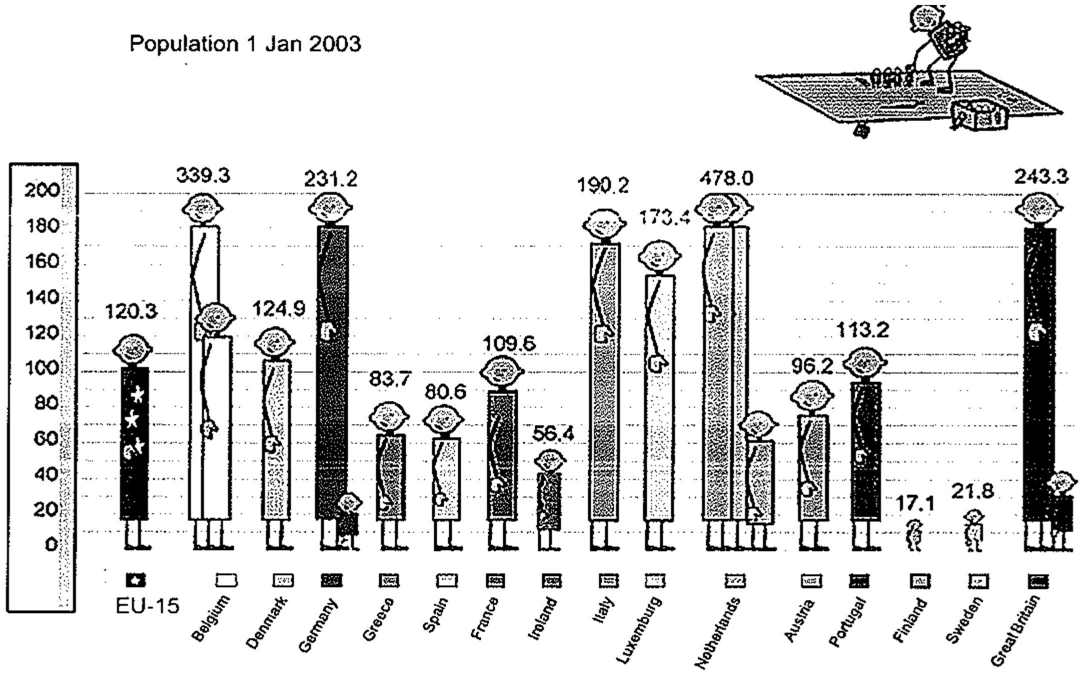
In addition, the task is supplemented by elaborate assessment criteria intended for summative assessment but also well suited as a foundation for formative assessment. The assessment is based on three aspects. The first aspect is understanding, and addresses to what extent the students show that they have understood the question, the concepts and relationships between them. The second aspect is language and assesses the clarity of the student's explanations, and how well they use mathematical language. The third aspect focuses on degree of participation, i.e. to what extent the students participate in the discussion, can argue for their ideas and respond to the explanation of other students.

Version B – EU's population (diagrams)



Source: Eurostat

Population 1 Jan 2003



Population density: inhabitants per square kilometre, 1 Jan 2003

Source: Eurostat

Version B – EU's population (statements)

The diagram shows that

1. the population of Spain is about 80 million.
2. Finland is the country with the lowest population density in EU.
3. EU has about 380 million inhabitants.
4. the country with the most inhabitants is also the most densely populated.
5. a little more than one tenth of the population of EU lives in Spain.
6. the three countries with the greatest number of inhabitants have, together, about 50 % of EU's population.
7. in Great Britain there is about twice as much area per person as there is in the Netherlands.
8. 5 % of EU's population lives in Sweden.
9. Portugal has 50 % more inhabitants than Finland.
10. in Sweden we have about 6 times more area per person than the average in EU.
11. the area of Spain is greater than that of Greece.
12. the area of France is almost double that of Italy.

The second example describes a model for working with oral competencies in mathematics rather than a specific mathematical task. This model has been developed as part of the assessment package for upper secondary school. It differs from the model presented above mainly through its exclusive focus on the oral competence. The model from comprehensive school presented above was partly about the oral proficiency of the students and partly about using oral presentations as a tool to learn about students understanding of the mathematical content. In the second Swedish example, each student is given a mathematical task of a kind that the individual student is most likely to solve with some understanding. The idea is that students should be able to solve the problem because the oral part is about how students can describe and reflect on their solution, not on revealing mathematical understanding in general. After working individually with a task, 4-5 students meet with the teacher and are given the opportunity to talk about what they have done. The other students in the group are encouraged to ask questions and engage in discussions. The teacher can also ask questions, but it is stressed that the idea of the group is to create a communicative environment where students primarily see their peers as the audience.

A number of mathematical tasks are suggested to the teacher, but these are not unique or highly designed for the purpose of oral communication. Teachers can use these tasks or find others that seem to fit what their students have been working with.

Comparison of length

The Swedish National Centre for Mathematics Education (NCM) at Göteborg University coordinates, supports, develops and implements the contributions which promote Swedish mathematics education from pre-school to university college. NCM has a website with lots of materials for teachers, including a resource based on the goals to strive for in the mathematics syllabus of the Swedish comprehensive school. The resource is constructed as a matrix with the goals to aim for as columns and the content-oriented goals to achieve in the rows. When you click on a cell in the matrix you will find relevant articles and also tasks and exercises relating to the particular combination of goals indicated by the position in the matrix.

One of the goals to aim for explicitly refers to communication, stating that students should develop their ability to understand, carry out and use logical reasoning, draw conclusions and generalise, as well as orally and in writing explain and provide the arguments for their thinking. The NCM material in this column of the matrix does not supply much that focuses on the communicative aspects of this goal. There are a couple of articles, and one or two tasks. One example is shown here.

In this task, students work in groups, and each group gathers around a table where the teacher has put 5-7 cards with different lengths printed. In collaboration, the students should order their cards, from the one with the shortest distance printed to the one with the longest. The students in the group must all agree on the order. When the groups are happy with their results, they change table and start working with another ser of cards. At the new table the group reflects on whether the presented order of cards is correct or if it should be changed. Suggestions for changes are noted on a piece of paper at this station. The groups continue to the next table and finally end up at the table where they started. In the final part of the task, the group read the suggestions from the other groups and take them into consideration.

Strävorna

4B Jämföra längd

... *Utvecklar sin förmåga att förstå, föra och använda logiska resonemang, dra slutsatser och generalisera samt muntligt och skriftligt förklara och argumentera för sitt tänkande.*

... *Olika metoder, måttsystem och mätinstrument för att jämföra, uppskatta och bestämma storleken av viktiga storheter.*

Exempel på en gruppuppsättning kort:

7 dm	2 m
35 cm	100 mm
1 dm 5 cm	2 mm

Written communication

Two tasks given in national tests in the final year of comprehensive school:

Lisa competes in archery. Each arrow can score at least 0 points and at most 10 points. At a competition Lisa shot 5 arrows. The mean score was 8 points and the median was 10 points. How might she have shot? Explain your choice and discuss various possibilities.

Language across the mathematics curriculum in Romania

Mihaela Singer

1. Mathematics

1. 1. The structure of mathematics teaching

Mathematics is taught as a distinct subject from the first grade onwards. There are 3-4 periods of mathematics per week from the 1st grade to the 5th grade, 4 periods from the 6th grade to the 8th grade and 2-4 periods from the 9th grade to the 12th grade, the latter depending on the specialised track followed by the students. (Periods mean teaching hours or classes, which can be about 45-50 minutes long.) In addition, students in compulsory education might choose a supplementary optional class per week, which is allocated in the curricular area of Mathematics and Science. The number of optional classes increases in upper secondary.

There are 102-136 periods of mathematics per year from the 1st grade to the 5th grade, 136 periods from the 6th grade to the 8th grade and 68-136 periods from the 9th grade to the 12th grade. Mathematics tasks given to pupils to prepare at home are compulsory. There is no official regulation regarding this issue. Homework is always written and the quantity depends on the teacher.

1.2. Teaching methods

The mathematical contents are prescribed at national level. There is a mathematics curriculum for pre-school education, as well as one for primary and secondary education.

The teaching methods are free to be developed by each teacher. The curriculum does not impose certain methods but provides *examples of learning activities* for each reference objective. Practical activities and problem solving are relatively important. There are objectives that deal with group-work activities but teachers still regard this as not being compulsory to use. Advanced classroom management (as using a certain type of class organisation as related to certain objectives) is known and used by a small number of teachers.

Different resources may be used in mathematics teaching: objects, shapes, drawings, computers, no calculators. The role of practical activities is very important in primary school, where pupils use different objects (sticks or marbles) in order to calculate. Later on there is an emphasis on the different instruments of measurement that they are going to use each year. Also, models of geometrical shapes are used to help them visualise abstract forms. The systematic use of computer software is relatively limited.

1.3. About textbooks

Since 1996, textbooks have been selected from a list of manuscripts in a national contest, which consists in assessing the content from the perspective of several criteria and a bid of costs. There is an officially approved (by the ministry) list of textbooks that have won the contest, and teachers are free to choose one among the approved textbooks. The selected textbook is the official textbook for pupils. In compulsory education, textbooks are paid for by the government. The mathematics textbooks are usually used in class as a source of exercises. Teachers might get teaching guides, which are not compulsory.

1.4. Mathematics assessment

Assessment includes tests, traditional exercises (oral and written), project work (seldom), self-assessment (seldom). Assessment is done on the basis of the objectives for each age level prescribed by the curriculum. The curriculum also contains curricular standards at the end of primary and lower-secondary school.

Teams of experts in curriculum and experts in assessment have developed band descriptors, which detail the curricular objectives on three levels. Usually, teachers are not familiar enough with the manner of using these levels and they are not aware enough of the basic knowledge children should posses at the end of each school year.

The first official examination to include mathematics takes place at the age of 15, at the end of the 8th grade. The role of this exam is to certify graduation from the compulsory schooling and give access to high schools. National assessments are conducted periodically on a representative sample at the end of the 4th grade.

2. Aspects involving language in the mathematics curriculum

The *National Curriculum* in Romania includes:

- *The National Curriculum for Compulsory Education. Framework of Reference* (regulatory document that ensures the coherence of the curricular system components, in terms of processes and products);

- *The Curriculum-framework plans for grades I-XII/XIII*, a document that establishes the curricular areas, school subjects and time allocations;

- *The Subject Curricula*, providing:

a) for grades I-VIII: the attainment targets, reference objectives, examples of learning activities, contents as well as the curricular standards of achievement;

b) for grades IX-XII: general competencies, specific competencies, with correlated relevant contents, values and attitudes, suggested methodology all of these being set for each school subject included in the Curriculum-framework plans;

- *Teachers' guides* describing ways to implement and monitor the curricular process.

The entire corpus of the *National Curriculum* documents were published and distributed in the system during the period 1997-2001. It totals over 50 volumes and more than 6,000 pages. There were two major changes in the maths curriculum for compulsory education in 1995 and 1999. Both led to changes in the formal curriculum, but did not greatly influence teaching practice. The last change in the official curriculum for mathematics in primary schools was introduced in 2003. This brought changes in the distribution of contents, not in the philosophy of the curriculum.

The *National Curriculum* in Romania is structured on seven curricular areas, which were assigned according to epistemological and psycho-pedagogical criteria. *The curricular area* offers a multi- and/or interdisciplinary view of school subjects. These curricular areas are: Language and Communication; Mathematics and Natural Sciences; Man and Society; Arts; Physical Education and Sports; Technologies; Counselling and Guidance. The curricular areas are the same for both compulsory schooling and high-secondary education, but their weight per key-stage and grade is variable.

The new dimensions within the curricular system for compulsory education stated in the documents are the following:

- place learning - as a process - in the centre of didactic approaches;
- orient learning towards the training of skills and attitudes, by developing problem solving abilities;
- put forward a flexible learning offer;
- adapt learning to everyday life as well as to students'+ needs, interests and aptitudes;
- introduce new ways to select and organise objectives and syllabi, according to the principle "not much, but well";
- open individualised school routes which motivate students by orienting them towards innovation and personal fulfilment;
- involve all educational actors in order to plan, monitor and assess the curriculum.

A close-up of Maths and Science in the National Curriculum could give a better view of the philosophy of this curriculum. Learning mathematics in the compulsory school system supposes understanding the nature of mathematics as an activity of problem solving based on a corpus of knowledge and procedures that can be approached by exploration, and also as a dynamic discipline, which is closely related to the society by its relevance in everyday life, in natural sciences, in technology and social sciences (Crisan and Singer, 1999). All these demand some major shifts in the way teachers think about their classroom activity. More specifically, these shifts are underlined in Table 1.

Table 1: Shifts in teaching and learning mathematics

What becomes less important:	What becomes more important:
memorising rules and computing	problem-solving activities involving trial-and-error, active involvement in practical contexts, search of solutions beyond the given frame of school knowledge
solving problems/exercises that have a unique answer	formulating questions, analysing the steps and motivating decisions in problem solving
'pen and pencil' (or 'chalk and blackboard') maths	using various manipulative activities to help learning
teacher acting as an information provider to a pupil that receives it passively and works alone	teacher acting as a facilitator of learning, stimulating pupils to work in teams
assessment with the purpose of labelling pupils	assessment as a part of learning, stimulating classroom activities

The framework objectives for mathematics in compulsory education are the following:

1. Knowledge and use of mathematical concepts
2. Development of exploration, investigation and problem-solving capacities

3. Communicate using mathematical language
4. Develop interest and motivation for studying mathematics and applying them to various contexts

The new philosophy of education promoted by the National Curriculum puts more emphasis on language across the curriculum. Thus, among the four framework objectives for mathematics in compulsory education, there is one devoted explicitly to communication.

Specific aspects that involve mathematics terminology - argumentation in mathematics, understanding text, textuality, identifying text types, genre, context, which are inherently involved in word-problem solving - are reflected indirectly in the curricular standards of achievement for primary and lower-secondary education. Tables 3 and 4 reproduce the basic level of these standards.

Table 2: The mathematics curriculum. Curricular standards of achievement for primary education

	Framework objectives	CURRICULAR STANDARDS OF ACHIEVEMENT (Basic level)
1	Knowing and using specific mathematical concepts, terminology and computing procedures specific to mathematics	S1. Write and read numbers up to 1,000,000 S2. Use mathematical terminology correctly S3. Perform addition and subtraction with natural numbers lower than 1,000,000 S4. Perform multiplication and division with natural numbers lower than 1,000 S5. Use fractions in simple exercises of addition and subtraction with the same denominators
2	Developing capabilities for exploration/ investigation and problem solving	S6. Recognize, represent and classify 2D and 3D shapes S7. Formulate and solve problems that involve performing at most three operations S8. Use arithmetic reasoning in problem solving situations S9. Use simple modalities to organize and classify data S10. Recognise and develop patterns for sequences S11. Perform estimations and approximations within practical situations S12. Use non-conventional measure units in various contexts S13. Use conventional measure units for time, mass, length, and volume of objects
3	Developing the capability to communicate using the mathematical language	S14. Express computing strategies and the results of exercises and problems in a concise and clear manner, verbally and in writing,

Table 3: The mathematics curriculum. Curricular standards of achievement for the end of compulsory education

	Framework objectives	CURRICULAR STANDARDS OF ACHIEVEMENT (Basic level)
1.	Knowledge and understanding concepts, terminology and computing procedures specific to mathematics	S.1 Write, read, and compare real numbers and represent them on an axis S.2 Perform operations with real numbers (possibly represented by letters) S.3 Use estimates and approximations of numbers and measurements (lengths, angles, surfaces and volumes) to appreciate the validity of results S.4 Use elements of logic and set theory, as well as relations, functions and sequences in solving problems S.5 Solve equations and inequations and perform algebraic calculations using algorithms, specific formulae and methods S.6 Establish and use qualitative and metric properties of geometric 2-D and 3-D shapes in problems involving demonstrations and computations S.7 Use the relative positions of geometric shapes and elements of geometric transformations S.8 Record, process and present data using elements of statistics and probabilities
2.	Developing the capacity to explore, research and solve problems	S.9 Identify a problem and organise its solving efficiently S.10 Use various representations and methods to clarify and justify (proof) statements S.11 Build generalisations and check their validity
3.	Developing the capacity to communicate using the mathematical code	S.12 Understand the overall significance of mathematical information from various sources S.13 Express one's own attempts to solve a problem correctly, orally or in writing, S.14 Engage in mathematics activities as a member of a group

To gain a better idea about the curriculum content and its relationship with language, I synthesised on a few tables the reference objectives for each of the four framework objectives in grades 1 to 4. For grades 5 to 8, I have synthesised in a table those reference objectives for the framework objective that explicitly involve communication. This organisation gives a better view of the progression intended in developing the students' competences along the study of mathematics in compulsory education. This overview is given in the attachment.

3. Some conclusions

Some conclusions about language across the mathematics curriculum in Romania can be drawn based on the Vollmer-Ongstad (2007) list. There is an explicit emphasis on language in the mathematics curriculum for compulsory education in Romania that takes into account different facets of language.

Language as direct communication, as a way to exchange ideas and interact with others, to jointly construct meaning in pairs, in small groups or in the classroom as a whole, to negotiate meaning. Among the four framework objectives, one is devoted to communicating using the mathematical language. The reference objectives for "Communicating using the mathematical language", as for the other framework objectives, are constructed in progression from grade 1 to the last grade of compulsory education.

Language as an expression of understanding and text comprehension. Some reference objectives emphasise reading and writing mathematical texts, as well as decoding mathematics texts through the help of logical operators and quantifiers.

Language as (disciplinary) content (especially basic meanings/terms and expressions). Language as reflecting the structure of a topic or theme is emphasised within the framework objective "Knowledge and use of mathematical concepts", in the learning activities that are focused on terminology.

Language as discursive pragmatics, language as realisations of basic discourse functions (like naming, defining, describing, explaining, supporting, reporting, hypothesising, evaluating, etc.) is emphasised in various examples of learning activities that are provided within the subject curriculum for each grade.

Aspects related to language as creativity (the rheme/new part in theme-rheme or given-new dynamics) language as the tool and means for developing, creating and expressing new concepts and insights are targeted through the framework objective "Developing competencies in exploration, investigation and problem-solving".

Language for reflecting (critically) on the subject and one's own learning is emphasised through the framework objective "Developing interest and motivation for studying mathematics and applying it to various contexts". The reference objectives focused on communication and on developing attitudes are fundamental for metacognition and aim at the very core of education in the 21st century.

References

Cerkez, M., Singer, F.M., Sarivan, L. (coordinators) (1999): *National Curriculum for lower secondary education. Programs of study*, Volumes 1-10, (in Romanian), Bucharest, Cicero Publishing House.

Crisan, Al., Georgescu, D., Singer, F.M. (coordinators) (1998): *National Curriculum for compulsory education. Curriculum framework*, (in Romanian), Bucharest, Corint Publishing House.

Singer, F.M. (coordinator) (1999): *The New National Curriculum*: *A Synthesis.* Bucharest, Prognosis Publishing House.

Singer, F. M., (2001): Structuring the information – a new way of perceiving the content of learning, *Zentralblatt für Didaktik der Mathematik/IRME, MATHDI*, 6, 204-217.

Singer, F. M. (coord.) (2001): *Methodological guide for Curricular area Mathematics and Natural Sciences.* Grade 9 to 12, Bucharest: Aramis Publishing House.

Singer, M., Voica, C. *Challenging the future: mathematics education in Romania between ideals and reality*, Cub, ICME-10, 2004

Vlasceanu, L. (coord.) (2003): *The school at crossroads – Change and Continuity in the Curriculum for Compulsory Education*, (in Romanian, with a synthesis in English), Bucharest: Polirom P.H.

Vollmer, H.; Ongstad, S. (2007): Some notes on crucial concepts/ Report from the LAC Mathematics Education Group (LAC/MEG). No date.

Appendix (Singer)

A. A few words about the structure and content of the Romanian education system

The structure of the education system

Compulsory education starts at 6-7 years old with the 1st grade. It ends at 16 with the 10th grade. At the end of lower-secondary school there is a national examination.

Table A presents the structure of the Romanian education system and the system of qualifications in Romania. Basically, the common understanding of *qualifications* bears a vocational meaning.

Table A: The present system of qualifications in Romania

<table>
<tr><th>Age</th><th>Grades</th><th>ISCED</th><th colspan="3">Educational level</th><th>Qualification level</th><th>Types</th></tr>
<tr><td rowspan="3">> 19</td><td rowspan="3"></td><td rowspan="2">6
5</td><td colspan="2">Post-University</td><td rowspan="2">Higher and post-
University education</td><td rowspan="2">5
4</td><td rowspan="3"></td></tr>
<tr><td colspan="2">University</td></tr>
<tr><td>4</td><td colspan="2">Post-high school</td><td>Post
secondary education</td><td>3</td></tr>
<tr><td>18</td><td>XIII</td><td rowspan="3">3</td><td></td><td rowspan="2">High school-upper cycle</td><td rowspan="3">Upper secondary education</td><td rowspan="2">3</td><td rowspan="3">Post-compulsory</td></tr>
<tr><td>17</td><td>XII</td><td rowspan="2">High school
upper cycle</td></tr>
<tr><td>16</td><td>XI</td><td>Completing year</td><td>2</td></tr>
<tr><td>15</td><td>X</td><td rowspan="4">2</td><td rowspan="2">High school
lower cycle</td><td rowspan="2">Arts and crafts school</td><td rowspan="6">Lower secondary education</td><td rowspan="2">1</td><td rowspan="10">Compulsory</td></tr>
<tr><td>14</td><td>IX</td></tr>
<tr><td>13</td><td>VIII</td><td colspan="2" rowspan="4">Lower secondary school (Gymnasium)</td><td rowspan="11"></td></tr>
<tr><td>12</td><td>VII</td></tr>
<tr><td>11</td><td>VI</td><td rowspan="6">1</td></tr>
<tr><td>10</td><td>V</td></tr>
<tr><td>9</td><td>IV</td><td colspan="2" rowspan="4">Primary school</td><td rowspan="4">Primary education</td></tr>
<tr><td>8</td><td>III</td></tr>
<tr><td>7</td><td>II</td></tr>
<tr><td>6</td><td>I</td></tr>
<tr><td>5</td><td></td><td rowspan="3">0</td><td colspan="2" rowspan="3">Pre-school education</td><td rowspan="3">Pre-school education</td><td rowspan="3"></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td>3</td><td></td></tr>
</table>

Primary and secondary teachers and class organization

In grades 1-4, one teacher teaches all subjects for a class with some exceptions (languages and religion are taught by specialised teachers). A primary teacher usually teaches 18 hours per week. Beside teaching hours, he or she is required to be in school for various other activities – but there is no specified time load. During a school year a primary teacher works with a single class, which s/he teaches for several successive years. There are about 25 pupils of the same age in a class.

In lower secondary, each subject is taught by a single specialised teacher. The teacher's workload is 16-18 hours per week.

Concerning pre-service and in-service training:

Until 1999, primary teachers were trained in pedagogic high-schools, which provided them with a mathematics training similar to any other humanistic high school, plus a course in arithmetic and one in mathematics teaching. Since 1999, initial training of primary teachers is done in 3-year university colleges. Mathematics training in these colleges is similar to that in pedagogical high schools.

Mathematics teachers for secondary education used to be trained at university level for 4-5 years. Since 2005, within the Bologna process, university training takes 3 years. In addition, to get a teaching certificate, graduates must enrol for the psycho-pedagogical module.

In-service training for teachers is organised through Pedagogical Colleges and county Teachers' Centres. They have a formal training offer for every year.

Concerning recruiting:

The local education inspectorates recruit the schoolteachers. The recruitment method depends on the type of position within the school; there is usually a contest at the county level in which the CV is assessed under a set of criteria. The head teacher of the school is not involved in the recruitment.

Concerning the teachers' appraisal:

Currently, there is a teachers' appraisal as result of an inspection or as result of the periodical assessments they have to pass (every 5 years).

Inspectors are responsible for teacher's appraisal, while the head of school may give advice. The school head teacher also makes a formal appraisal at the end of each school year.

B. Progression for objectives in mathematics education in Romania

Tables B - E below present the progression in developing specific objectives for mathematics in primary education (Grades 1 to 4).

Table B: Progression in developing the reference objectives for the mathematics curriculum. Framework objective 1:

Knowledge and use of mathematical concepts

Reference objectives for Grade 1	**Reference objectives for Grade 2**	**Reference objectives for Grade 3**	**Reference objectives for Grade 4**
At the end of grade 1, students will be able to:	*At the end of grade 2, students will be able to:*	*At the end of grade 3, students will be able to:*	*At the end of grade 4, students will be able to*:
1.1. understand the place-value system for 2-digit numbers with the help of objects	**1.1.** understand the place-value system for 3-digit numbers with the help of objects	**1.1.** know and use the meaning of the position of digits in writing positive integers less than 1,000	**1.1.** know and use the meaning of the position of digits in writing positive integers up to and including the billion magnitude
1.2. write, read and compare positive integers between 0 and 100; *use symbols ‘<’, ‘>’, and ‘=’ *to compare numbers correctly*	**1.2.** write and read positive integers between 0 and 1,000, and compare positive integers smaller than 1,000, using the symbols: ‘<’, ‘>’, and ‘=’	**1.2.** write, read, compare and sort positive integers up to 1,000,000	**1.2.** write, read, compare and sort positive integers
1.3. add and subtract: - **numbers between 0 and 20 using carrying out techniques* - numbers between 0 and 30; no carrying out techniques - **numbers between 0 and 100, no carrying out techniques*	**1.3.** add and subtract: - numbers between 0 and 100 without and with carrying out techniques -**numbers between 0 and 1000*	**1.3.** add and subtract numbers less than 1,000	**1.3.** use fractions for expressing subdivisions of the integer
	1.4. perform: multiplication with numbers between 0 and 100, using repeated addition or the multiplication table up to 50; division with numbers less than 50 by repeated subtraction or as a check for multiplication; **multiplication and division with numbers between 0 and 100*	**1.4.** perform multiplication with numbers between 0 and 1,000, using the multiplication table or properties of multiplication	**1.4.** understand the significance of arithmetic operations and the use of algorithms for addition, subtraction, multiplication and integer division of positive integers
		1.5. perform division of a number less than 100 by a 1-digit number; **to divide a 3-digit number by a 1-digit number* - **to divide a 3-digit number to one digit number*	**1.5.** understand the meaning of addition and subtraction of fractions, to perform additions and subtractions of such numbers
		1.6. estimate the magnitude of the result of a one-operation exercise, by rounding up the numbers involved, with the aim of discovering possible mistakes	**1.6.** estimate the magnitude of the result of a one-operation exercise, by rounding up the numbers involved, with the aim of discovering possible mistakes
1.4. recognize 2-D and 3-D shapes, sort and classify objects according to their shape **1.5.** identify relative positions of objects in space; place objects in various relative positions	**1.5.** recognize 2-D and 3-D shapes and classify objects according to their shape **1.6.** identify relative positions of objects in space; place objects in various relative positions	**1.7.** sort and classify objects and drawings according to their shape; observe simple symmetry properties on drawings	**1.7.** recognise 2-D and 3-D shapes, identify and describe simple properties of 2-D shapes

1.6. measure and compare lengths, capacities and weights of objects, using non-standard simple units accessible to children; recognise hours on the watch	**1.7.** measure and compare lengths, capacities and weights of objects, using adequate non-standard units as well as the following standard units: meter, centimeter, liter **1.8.** use units for time and money	**1.8.** know the standard units for length, volume, weight, time, and money and use transformations between these units and their respective multiples and submultiples	**1.8.** know the standard units for length, volume, weight, surface, time and money units and use transformations between multiples and submultiples of the same unit

Table C: Progression in developing the reference objectives for the mathematics curriculum. Framework objective 2:

Development of exploration, investigation and problem-solving capacities

Reference objectives for Grade 1	Reference objectives for Grade 2	Reference objectives for Grade 3	Reference objectives for Grade 4
At the end of grade 1 students will be able to:	*At the end of grade 2 students will be able to*:	*At the end of grade 3 students will be able to*:	*At the end of grade 4 students will be able to*:
2.1 explore ways of writing numbers smaller than 20 as a sum or a difference; * *explore ways of writing numbers smaller than 100 as a sum or a difference* **2.3** estimate the number of objects in a set and check the estimation by counting	**2.1** explore various ways of decomposing numbers smaller than 100 **2.2** estimate the magnitude of a result of an operation in order to limit computing errors	**2.1** explore ways of decomposing positive integers smaller than 1,000, using any known operation **2.2** perform integer division of a number by a 1-digit number and link it to the formula dividend = divisor x quotient + rest, rest < dividend, by using repeated subtraction or multiplication	**2.1** explore ways of decomposing positive integers less than 1000, using any of the four arithmetic operations or their combinations **2.2** estimate the truth of an assertion and know the sense of the implication "if-then" for simple, everyday examples
2.2 observe the correspondence between the elements of two sets of objects, drawings or positive integers smaller than 20	**2.5** observe the correspondence between the elements of 2 different categories of objects (sequences, numbers less than 100), based on given rules; continue repetitive models represented by objects or numbers less than 100; - * *create sequences using given rules*	**2.3** discover, recognize and use patterns in sequences of objects or numbers, composed using given rules **2.4** use symbols for replacing unknown numbers while solving problems	**2.3** discover, recognize and use patterns in sequences of objects or numbers, composed using various rules **2.4** use symbols for replacing unknown numbers while solving problems
2.4 solve problems involving one operation (addition/subtraction) **2.5** device exercises and problems with numbers between 0 and 20, orally	**2.3.** solve problems that require one operation from those already studied - **solve problems that require at least 2 operations of addition or subtraction* **2.4** compose exercises and problems with numbers between 0 and 100 that involve one operation, orally	**2.5** solve and compose text problems of the following types: $a \pm b = x$, $a \pm b \pm c = x$, a x $b = x$, $a : b = x$, $b \neq 0$, where a, b, c are given positive integers less than 1,000, and x is an unknown number	**2.5** solve and compose text problems
	2.6 extract information from tables and lists, collect data by observation on a certain theme, represent data in tables	**2.6** collect data, sort and classify them on simple criteria, organise these data in tables	**2.6** collect data, sort and classify them on simple criteria, represent these data in tables **and give simple interpretations*

Table D: Progression in developing the reference objectives for the mathematics curriculum. Framework objective 3:

Communicate using the mathematical language

Reference objectives for Grade 1	Reference objectives for Grade 2	Reference objectives for Grade 3	Reference objectives for Grade 4
At the end of grade 1, students will be able to:	*At the end of grade 2 students will be able to*:	*At the end of grade 3 students will be able to*:	*At the end of grade 4 students will be able to*:
3.1. verbalize constantly computation patterns	**3.1.** express his or her own way of solving a problem (the steps) orally or in writing, using words	**3.1.** express clearly and concisely the meaning of calculations done in order to solve a problem	**3.1.** develop a plan, orally or in writing, to explain his or her approach in solving a problem

Table E: Progression in developing the reference objectives for the mathematics curriculum. Framework objective 4:

Develop interest and motivation for studying mathematics and applying it to various contexts

Reference objectives for Grade 1	Reference objectives for Grade 2	Reference objectives for Grade 3	Reference objectives for Grade 4
At the end of grade 1 students will be able to:	*At the end of grade 2 students will be able to*:	*At the end of grade 3 students will be able to*:	*At the end of grade 4 students will be able to*:
4.1. manifest availability and pleasure for using numbers.	**4.1.** manifest curiosity for finding out the results of exercises and problems	**4.1** manifest initiative in proposing various approaches in solving a problem	**4.1.** manifest interest for analysing and solving practical problems through mathematical methods
		4.2. demonstrate a proper behaviour towards colleagues in the workgroup during practical problem-solving activities	**4.2.** overcome obstacles in problem solving, use trial-and-error to find new ways of solving a problem
			4.3. manifest availability in learning from others and in helping others in problem solving activities

The progression in students' competence for communication within mathematics curricula for lower secondary education is emphasised in table F.

Table F: Progression in developing the reference objectives for the mathematics curriculum. Framework objective 3: Communicate using mathematical language

Reference objectives for Grade 5	Reference objectives for Grade 6	Reference objectives for Grade 7	Reference objectives for Grade 8
At the end of grade 5 students will be able to:	*At the end of grade 6 students will be able to:*	*At the end of grade 7 students will be able to:*	*At the end of grade 8 students will be able to:*
3.1. identify essential information from a mathematical text having various presentations	**3.1.** differentiate information from a mathematical text taking into account their nature	**3.1.** identify and differentiate the phases of a mathematical proof	**3.1.** find information of mathematical nature in a variety of sources and understand their global meaning
3.2. show some methods/operations used to solve problems clearly, correctly and concisely, in oral or in written form	**3.2.** show the succession of operations in problem solving clearly, correctly and concisely, in oral or in written form, using adequate terminology and mathematical notations	**3.2.** display the solution of a problem coherently, using various expressing modalities (words, mathematical symbols, diagrams, tables, constructions made from various materials)	**3.2.** display the solution of a problem coherently, correlating a variety of expressing modalities (words, mathematical symbols, diagrams, tables, constructions made from various materials)
3.3. assume learning roles in a group	**3.3.** discuss about the correctness of a mathematical approach, bringing arguments for his/her opinions within a group	**3.3.** sustain ideas and mathematical methods within a group, use various information resources in checking and constructing arguments	**3.3.** discuss about the advantages/ disadvantages of a certain solving method or of a certain mathematical approach, within a group

Language and communication in Norwegian curricula for mathematics

Sigmund Ongstad

Language in the part 'Objectives'

In this examination of where 'language' might be found in mathematics curricula, a differentiation will be made between explicit and implicit, and between language in a narrower and a broader sense (Ongstad, 2006a). The national curriculum from 2006, LK06, will be given priority, especially focusing on the end of compulsory education/schooling (year 10). In addition, some visits will be paid to the 1997 curriculum (L97) and to other stages/years in LK06.

The LK06 curriculum in mathematics starts with giving *objectives* for the subject over ca. 30 lines of text (Utdanningsdirektoratet, 2007). Only one of these lines (here underlined) concerns language and communication explicitly: *Problem solving is part of mathematical competence. This means analysing and transforming a problem into mathematical form, solving it and assessing the validity. <u>This has linguistic aspects, such as reasoning and communicating ideas</u>* (Utdanningsdirektoratet, 2007:1). To find just one single reference is quite unexpected, given the strong weight language-oriented learning theories, stemming both from both Piaget and Vygotsky, have had in mathematics education internationally and in Norway, at least on the rhetorical level.

If we broaden the scope and read the whole chapter again, now in the light of semiotics and more general concepts and conceptions such as 'culture' and 'lifeworlds', it becomes clear that the view of mathematics expressed in this curriculum is in some sense nevertheless rather broad. There is explicit mentioning of (...) *the joy people have felt when simply working with mathematics.* Mathematics is further seen as an important part of culture and it is claimed that (...) *the subject plays a key role in general education by influencing identity, thinking and understanding of oneself.* Mathematics is even seen as important for 'design', a concept that has grown in importance lately. In this perspective, to analyse forms and structures is seen as crucial.

One can conclude so far that, judging from the introduction, an *explicit* view of the importance of language plays no significant role for the new national curriculum for mathematics in Norway. But one cannot, therefore, conclude that this school subject is perceived narrowly, just focusing on its own disciplinarity. Mathematics is seen as culturally integrated in people's lifeworlds. This wider horizon is important, when extending the perspective from language as explicit, to semiotics and communication as implicit, given the definition of semiotics as "the study of *cultural* utterances as meaning".

Language in the subject areas

The concrete main subject areas for years 1-10 (ages 6-16) in LK06 are: Numbers/and algebra, geometry, measuring, statistics/and probability and combinatorics and function. Hence there is no explicit mentioning of language, semiotics or communication in this second part of the curriculum, which defines its content. Neither are there any implicit hints to such dimensions. Only for the last year in upper secondary, year 13, is there a single subject area, called *culture and modelling*, that might be related (implicitly) to semiotics.

Language and 'Basic skills'

One of the most disputed parts in the LK06 national curriculum is a chapter called *Basic skills* (UD, 2007:4). These are mandatory for all school subjects in the written curriculum, not just mathematics. In the version cited here, the national committee for the mathematics curriculum has put in concrete terms what 'basic skills' implies for mathematics. Being important for understanding the whole curricular enterprise, the new national reform, the original text is quoted in full (this is the Ministry's official English version):

Basic skills are integrated in the competence aims where they contribute to development of the competence in the subject, while also being part of this competence. In the mathematics subject the basic skills are understood as follows:

> *Being able to express oneself orally* in the mathematics subject means making up one's mind, asking questions, reasoning, arguing and explaining a process of thinking using mathematics. This also means talking about, communicating ideas and discussing and elaborating on problems and solution strategies with others.
>
> *Being able to express oneself in writing* in the mathematics subject means solving problems by means of mathematics, describing a process of thinking and explaining discoveries and ideas; one makes drawings, sketches, figures, tables and graphs. Furthermore, mathematical symbols and the formal subject language are used.
>
> *Being able to read* in the mathematics subject means interpreting and using texts with mathematical content and content from everyday life and working life. Such texts may include mathematical expressions, graphs, tables, symbols, formulas and logical reasoning.
>
> *Being able to do mathematics* in the mathematics subject is, needless to say, the foundation of the mathematics subject. This involves problem solving and exploration, starting with practical day-to-day situations and mathematical problems. To manage this, pupils must be familiar with and master the arithmetic operations, have the ability to use varied strategies, make estimates and assess how reasonable the answers are.
>
> *Being able to use digital tools* in the mathematics subject involves using these tools for games, exploration, visualisation and publication, and also involves learning how to use and assess digital aids for problem solving, simulation and modelling. It is also important to find information, analyse, process and present data with appropriate aids, and to be critical of sources, analyses and results.

From almost no connections to language and communication in the general introduction and in the subject content, part this chapter all of a sudden turns the relationship between mathematics and language more or less 'upside down'. So far in the curriculum no signal has been given to the reader of any 'integration' of the two discourses, that we may conceptualise as 'disciplinarity' related to mathematics and 'discursivity' related to language and communication. There should be no doubt that all the writing teams of this national curriculum have been instructed to spell out what the three first basic skills will imply for their particular subject.

While the expressions *Being able to express oneself orally/in writing* (etc.) are general and originally the same for all school subjects, the further description of each skill is accordingly disciplinary and specific. Hence, what one may consider as an educational ideology of communication is here *imposed* upon representatives of a discipline or school subject by the ministry. The skills are defined and presented for the curricular groups irrespectively of how these might be understood in a broader sense as communication or semiotics.

Language in 'competence aims' at different stages

The rest of the written curriculum is structured as a series of bullet points for aims in each of the five main subject areas to which the mathematics curriculum has given priority (all mentioned above). These are supposed to be achieved after year 2, year 4, year 7 and year 10. For year 10, which is the main focal year for our inspection, there are a total of 24 points (for the five main areas in all) that spell out what pupils shall be able to at different stages. None of these aims has any explicit mention of the role of language or communication. The closest is probably the formulation (...) *describe sample space and express probability as fractions*.

Integration or schism?

One can conclude that the new national curriculum for mathematics in Norway (LK06) in use from 2006 onwards, gives absolute priority to disciplinarity in the three parts 'objectives', 'subject areas' and 'competence aims'. In the fourth part, 'basic skills', language and communication, that is, 'discursivity', is just as dominant. *This schism is the plan's most significant pattern*. This creates an uncertainty of what the main intention is: is it the objectives or is it the skills? The skills chapter can also be read as a way of 'forcing' mathematics (and the other school subjects in the curriculum) to be *tools* for, or to mediate, a wider enculturation (or perhaps even 'Bildung') rather than being an isolated, purely disciplinary knowledge.

Methods and approaches?

It should be added that the 'lack' of a chapter on methods and approaches is the result of a deliberate political and didactic choice the ministry has made. In such a chapter one could expect to find ideas about how the tension between a mathematical and a communicative approach could be resolved. A practical consequence of leaving out this traditional part of a curriculum is that all schools/municipalities are expected to make their own, local curricula in which such approaches are made explicit. [Regarding the possibility of inspecting actual priorities of values expressed rhetorically in the plan, the first sets of exams will probably not be available before spring 2008. Also, there are few textbooks out yet that can indicate how this 'empty space' might be filled.]

No role for language at the start?

If we compare the findings from this rapid 'investigation' with what the 13 competence aims are for/after the second year, we find that not even at this level is language given any significant role (here I have left out the four main domains). The closest is probably rather vague formulations such as *talk about*, *describe them orally* and very vaguely *name days* etc.

The aims for the education are that the pupil shall be able to

- count to 100, divide and compose amounts up to 10, put together and divide groups of ten
- use the real number line for calculations and demonstrate the magnitude of numbers
- make estimates of amounts, count, compare numbers and express number magnitudes in varied ways
- develop and use various arithmetic strategies for addition and subtraction of double digit numbers
- double and halve
- recognise, talk about and further develop structures in simple number patterns
- recognise and describe characteristics of simple two- and three-dimensional figures in connection with corners, edges and surfaces, and sort and name the figures according to these characteristics
- recognise and use mirror symmetry in practical situations
- make and explore simple geometrical patterns and describe them orally
- compare magnitudes for length and area using suitable measurement units
- name days, months and simple times of day
- recognise Norwegian coins and use them when buying and selling
- collect, sort, note and illustrate simple data with counting lines, tables and bar graphs

Hence, not even as an 'initiation' to the subject is language and communication given any significant role in this curriculum.

What role is mathematics given in the general *Core Curriculum*?

One should know that the *general* curriculum for LK06, also called the *Core Curriculum*, has been accepted and used, almost without any changes, by shifting governments over a broad political spectre for the last 13 years (TRMERCA, 1999). It describes what may be called Bildung-like elements, as six different 'human beings', characterised by the following adjectives 'spiritual', 'creative', 'working', 'liberally-educated', 'social' and 'environmentally aware' (... human beings). These aspects are supposed to be brought together in *the integrated human being* (which some interpreters see as a seventh human being).

However, this chapter in LK06 explicitly warns that an integration is inevitably *dilemmatic* in its nature. 17 concrete oppositions are described to illustrate this challenge. Furthermore, there is nothing in the 17 dual aims formulated in the chapter *The integrated human being* that directly makes room for the overall aims in mathematics in LK06. The 17 dilemmatic 'aims' are supposed to problematise and raise awareness of how aims within and between different school subjects can actually be in conflict when one attempts to integrate them.

While one can easily find many roles and important connections for all the other school subjects in this overall and (over-?)ambitious educational enterprise, mathematics has a very weak position in this (con-)text, if any. One of the few places it actually is mentioned is under 'the creative human being'. Here, three main traditions are mentioned (practical, theoretical and cultural): *Learners meet the theoretical tradition in subjects where new knowledge is won through* _theoretical development_*, tested by logic and facts, experience, evidence and research. It is presented in the study of languages, mathematics, social and natural sciences* (RMERCA, 1999:29). The text is accompanied by an illustration of a page from the writings of Pythagoras in (old) Greek.

If this very loose (or almost non-existent?) connection between mathematics and the general curriculum reflects a general situation, mathematics education seems to have severe problems developing convincing didactic arguments for in which sense and to which degree it actually intend to contribute to 'Bildung' or to 'the integrated human being'. It is not necessarily so that it does not, but the didactic responsibility of making this explicitly aware for students and other readers seems not to be adequately met.

One can therefore even speculate whether the seemingly strong position mathematics has had, both on a rhetorical level in society and in the mind of students and parents, may actually be counter-productive to the idea of school subjects making students 'spiritual', 'creative', 'working', 'liberally-educated', 'social', 'environmentally aware' and finally 'integrated'.

The inspection of the mathematics section in the 2006 plan revealed that *integration* of communicational and disciplinary dimensions is almost no-existent. This pattern seems to be in line with a general impression of mathematics as more 'isolated' relative to curricular ambitions to create Bildung (Ongstad, 2006b). Hence there might be good reasons for doubting the will in the field of mathematics education to see mathematics as a means and rather as an independent discipline.

Language and communication in the former curriculum, L97?

If we compare the curriculum for mathematics in L97 with the one in LK06, the main pattern is similarity, especially focusing on the format and the key curricular categories. However, if we look at the *introduction* in L97, where the subject and the educational aims are described, many formulations reflect a deeper and a more explicit understanding of the interface between mathematics and the lifeworld than we found in the 2006 plan: *Mathematics has many modes of expression and is undergoing constant development. It is a science, an art, a craft, a language and a tool* (RMERCA, 1999:165). And: *The syllabus seeks to create close links between school mathematics and mathematics in the outside world. Day-to-day experience, play and experiment help to build up its concepts and terminology* (RMERCA, 1999:165). Further: *Mathematical insight and skills are needed in order to understand and utilise new technology, and it is also a key to communication in modern societies* (RMERCA, 1999:165). Finally: *Familiarity with its language and symbols, and a clear grasp of its concepts, are important prerequisites for progress in mathematics* (RMERCA, 1999:166).

It is probably not fair to make a direct comparison between the two curricula, since the groups that wrote the 2006 plan were not 'allowed' to use more than one page on the general description of the subject. Besides, as mentioned, a (traditional) chapter on 'approaches' was left out (seemingly to allow for more local freedom). Nevertheless, the L97 had a conscious will to establish a *constructivist* view of the relationship between important aspects of language and mathematics that is more or less left behind in LK06: *Pupils have already developed some mathematical concepts when they start school although they may have difficulties expressing them in words*

(RMERCA, 1999:166). Also: *learners construct their own mathematical concepts. In that connection it is important to emphasise discussion and reflection* (RMERCA, 1999:167). It should be added, though, that the language conception we find here seems mainly connected to a traditional *conceptual* or basically a *semantic* view of which aspects of language are most relevant for the teaching and learning of mathematics.

Disciplinarity and discursivity as separated and the LAC approach

Formally, the descriptions of language and communication under *Basic skills* in LK06 are strong and explicit. But in general and in practice the separation is more or less total. Readers/teachers are left with few, if any, ideas about what a reasonable, valid and didactic relevant connection should or could be. This 'lesson' is very important for further CoE work on LAC for several reasons. Firstly, because there might even exist an 'alien' attitude to the idea of seeing mathematics as language in some circles, refuting the integration as a waste of energy. This position is often accompanied by a disciplinary pride in the many victories of mathematics, this universal language of real sciences that avoids the Babel of language.

Secondly, it will in any case be a challenge to persuade (communicatively) teachers and curriculum designers in mathematics who are positive to the importance of the connection, that it would be possible to find a sensible 'balance' of mathematics and language. Thirdly, it is, of course, not given which conceptual framework, which set of notions and perspectives, can be seen as adequate, given the many (relevant) disciplinary and cultural contexts for such a framework. Finally, the quite different disciplinary 'profiles' of teachers (Ongstad, 2006b) in different countries may influence, in different ways, to what extent an LAC approach can be introduced in primary and secondary mathematics education in Europe.

Conclusion?

An overall conclusion, then, can be that some agents/actors seem to want an *interwoven* curriculum, while others prefer to keep the dimensions/aspects *separate*. Some may not really want to interfere with 'language' other than as practical communication. If there is any significant 'developmental line' from 1997 to 2006 on this matter, a conceptual orientation seems lost. But a formal 'communicative' approach seems to have won a pyrrhic (formal) victory, and just in isolated parts of the curriculum. However, if future evaluations will focus on the basic skills rather than the detailed mathematical skills, the communication-related ambitions might be of practical relevance.

References

Ongstad, S. (2006a) Mathematics and Mathematics Education - Language and/or Communication? Triadic Semiotics Exemplified. *Educational Studies in Mathematics,* 61/1-2.

Ongstad, S. (2006b) Teacher of mathematics or Teacher Educator? Positionings and problematisations. In

Hudson, B. and J. Fragner (Eds.) *Researching the Teaching and Learning of Mathematics II* . Linz: Trauner.

TRMERCA (1999) *The Curriculum for the 10-year Compulsory School in Norway.* Oslo: The Royal Ministry of Education, Research and Church Affairs (L97).

Utdanningsdirektoratet (2007) *Mathematics Subject Curriculum.* (LK06). Visited 31.05.07.

www.udir.no/.../Fastsatte_lareplaner_for_Kunnskapsloeftet/english/Mathematics_subject_curriculum.rtf

Culture, language and mathematics education: aspects of languages in English, French and German mathematics education

Birgit Pepin, University of Manchester, United Kingdom

Abstract

This paper analyses the ways in which the particular culture of the mathematics classroom and the culture at large influence language in mathematics education in three countries: England; France; and Germany. Drawing on two recent comparative studies of mathematics education in the three countries, the findings of the research demonstrate that ideas, beliefs and principles, and every system's cultural and philosophical traditions, penetrate the educational enterprise in terms of language forms. It is argued that language in mathematics education needs to be understood in terms of the larger cultural context, and that an understanding of the wider meaning of language forms in a particular context can enhance communication between students and teachers, as well as between those involved in mathematics education across countries.

Introduction

Although language and communication factors have long been recognised to significantly influence mathematics teaching and learning, the question of how thought and language are related, and how this varies in different cultural systems, is currently been re-examined. Lean, for example, studied indigenous counting systems of Oceania, Polynesia and Melanesia. He found out that every distinct language has an associated unique counting system. Furthermore, he asserts counting systems are an integral part of language and that language is inextricably bound to culture (Lean, 1992 and 1995). Harris (1987) investigated measurement concepts in Aboriginal communities in Australia. She supports the view that aspects of indigenous mathematics, language and culture are inseparable. Other scholars, such as Zevenbergen (1995) and Stephen Harris (1990), also emphasise the importance of locating education theory and practice within cultural framework, and they distinguish between 'European' and 'Aboriginal' mathematics and culture. Harris (1990) claims that

> "the nature and degree of the difference between Aboriginal and European culture is so great that the only honest conclusion we can arrive at is that they are largely incompatible. … The degree of difference is so great that it is harder to find what they have in common in cultural terms than it is to see the differences."
>
> (Harris 1990, p.9)

Thus, whilst comparisons have been made between Aboriginal and European cultures, both are in themselves viewed to be homogeneous. What are the characteristics of a so-called 'European' culture? No distinction is made between the different European countries' cultural and educational traditions. This paper helps to fill that gap. Differences of language in mathematics education are identified in the English, French and German context. Furthermore, whilst it is re-emphasised that culture influences mathematics teaching and learning, and in particular mathematics language and communication, it is claimed that there are differences between English, French and

German cultural traditions which help to understand the ways in which mathematics is communicated in classrooms.

Culture, language and mathematics education

Mathematicians and mathematics educators communicate using language; they use it to teach mathematics, to share their understandings and to clarify and test understanding. Von Glasersfeld points out that communication is not as straightforward as one might assume.

> "Educators have spent and are rightly spending much time and effort on curriculum. That is, they do their best to work out what to teach and the sequence in which it should be taught. The underlying process of linguistic communication, however, the process on which their teaching relies, is usually simply taken for granted. There has been a naïve confidence in language and its efficacy. Although it does not take a good teacher very long to discover that saying things is not enough to 'get them across', there is little if any theoretical insight into why linguistic communication does not do all it is supposed to do."
>
> (von Glasersfeld 1983, p.43)

Bishop (1992) challenges the naïve view of the curriculum as an instrument for instruction regardless the cultural perspective.

> "The cultural perspective requires us to culturalise the curriculum at each of the levels, and demonstrates that no aspect of mathematics teaching can be culturally neutral. The cultural 'messages' in the educational enterprise are created and manifested by people. People create the national and local curriculum statements, people write the books and computer programs, people bring their cultural histories into the classroom, and people interpret and reconstruct the various messages."
>
> (Bishop 1992, p.185)

This confirms that people, with their beliefs, principles and practices that are underpinned by the 'culture' of the individual system or country, are central to the educational enterprise. Mathematical knowledge is constructed, interpreted and mediated by people, and language is part of communicating mathematical knowledge.

In mathematics classrooms, as in other subject lessons, language is not limited to the production of spoken or written texts, but it also includes other communication forms, such as gestures, board representations and charts, for example. While speaking or writing, non-verbal communication is used, intentionally or unintentionally. Both verbal and non-verbal communication often reflects the beliefs and thoughts of those who try to communicate. Thus, language can be regarded as an indicator and symbol of the belief system of the communicator's actions. However, patterns of communication are different in different 'cultures' and nation countries, because they are 'made of' and contain elements of traditions and practices that are linked to cultural and socio-economic conventions within each country or societal group. This helps to place the role of resource material, such as textbooks, and pedagogical practices into the way one communicates mathematics and convey its meaning and importance within every education system.

This paper explores communication structures used in mathematics classrooms in England, France and Germany. Essentially, it reports on findings emerging from research on mathematics teachers' work in the three countries and from research on mathematics textbooks and their use in English, French and German mathematics classrooms. Hundreds of hours of lesson observation were recorded in mathematics classrooms of the three countries. This article comes out of the context of these two comparative studies of mathematics teaching and learning in the three countries, and is not a result of an intensive language-based piece of research where the focus of these observations was language in the mathematics classroom. Instead, the data of the two comparative studies were re-examined with the focus on the use of language in mathematics education and the ways mathematical meanings were expressed by teachers and textbooks of the three countries. Furthermore, selected differences are examined and explained with regard to the cultural context and educational traditions of each individual country.

The language of teaching

> "Observing means interpreting; experience is interpreted through the patterns of knowledge and the value systems that are embodied in culture and in language."
>
> (Halliday 1978, p.203)

Halliday sees language in a socio-semiotic perspective, that is to say that he, firstly, refers to a culture, or social system, as a system of meaning. Secondly, he is concerned with the relationships between language and social structure (as one aspect of the social system), thus language is understood in its relationship to social structure. His rationale for choosing this particular angle is the following.

> "Learning is, above all, a social process; and the environment in which educational learning takes place is that of a social institution, whether we think of this in concrete terms as the classroom and the school, with their clearly defined social structures, or in the more abstract sense of the school system, or even the educational process as it is conceived of in our society. Knowledge is transmitted in social contexts, through relationships, like those of parent and child, or teacher and pupil, or classmates, that are defined in the value systems and ideology of the culture. And the words that are exchanged in these contexts get their meaning from activities in which they are embedded, which again are social activities with social agencies and goals."
>
> (Halliday 1985, p.5)

Thus, he puts language, context and text together. The notions of 'context of situation' and of 'context of culture' are both derived from Malinowski's writings, and both notions are necessary for an adequate understanding of the text. In essence, they say that the language is all part of the immediate situation and derives its meaning from the context in which it is used. In the following, examples are given of how language in mathematics education is influenced by the context of particular countries.

The history of mathematics has shown that the language of mathematics has developed, and is slowly changing as we speak, in order to meet social and individual needs. Teachers and students are likely to play a small part in this development. One example of the changes over time is the use of signs in English, French and German classrooms. Both in France and England, the times sign 'x' is used to indicate multiplication, whereas in Germany this is signified by a dot. This can be historically explained. In 1631 the English priest William Oughtred introduced the sign ‘x’ for ‘times’, whereas the common signs in Germany for multiplication (‘.’) and for Division (‘:’) go back to the scholarly writings of Gottfried Wilhelm Leibniz in 1693. Another important influence on German mathematical representation was the work of Adam Riese in his *Rechenbuch* of 1522.

Mathematical language registers are likely to respond to the needs of a changing system or society. For example, metre, centimetre, millimetre are words that are commonly used and understood as means for measurement of distances. The degree to which they are understood is however different in the three countries. In England, children often do not have a notion of metres, centimetres and millimetres, because they still use inches, feet and yards in their everyday language at home. This is only slowly changing. In Germany and France, this is different because they have never used imperial measurements. In France, even the ‘decametre’ is relatively commonly used, and taught in schools.

In some languages, written mathematics can reveal patterns that are not recognisable in the aural form. For example, the French expression for 80 is *quatre-vingts* (four twenties), and for 96 is *quatre-vingt-seize* (four twenties and sixteen). It must be relatively difficult for young French children just to write down the numbers, because every time small calculations are needed. English numbers read comparatively easy: *eighty* for 80 and *ninety-six* for 96, straightforward reading from left to right. However, this is not true for numbers between 10 and 20. In fourteen, for example, the four is read first, and then the ten. This is similar to the ways numbers are written and read in German. For example, 96 is spoken as *sechsundneunzig* (six and ninety). That means that a pupil writing down the number, first has to listen to ‘the end of the number’, before being able to write it down. But 127 is spoken as *einhundertsiebenundzwanzig* (one hundred seven and twenty), starting with the hundred, followed by the digits, and then the tens. How confusing! The point to be made here is that it is necessary for teachers to be sensitive to the language of mathematics and its sometimes arbitrary nature. The composition of numbers may cause confusion in the minds of young children and subsequently prevent them from focusing on and understanding the notions teachers are trying to teach.

In terms of pedagogy, a typical situation in a French classroom is the following:

> Teacher: “What is a *médiatrice*?”
>
> Pupils are hesitant, don’t know what to answer exactly, because they know that the teachers wants the definition that they had learnt previously.
>
> Teacher: “This is stuff from last year’s programme, and we have recently revised it. … Can’t you remember? Look in your *cahier de cours*.”
>
> Pupil reads and recites the definition of a *médiatrice*.

This situation is characteristic for France (Pepin, 2002). Mathematical terms are taught and expected to be learnt and their definition recited by pupils. Pupils often have to look up these terms, because they have difficulties with these ‘foreign’ words and cannot remember their meaning. The *médiatrice* is one of the four *droites remarquables dans un triangle*, one of the four ‘notable lines in a triangle’: *médiane* (side bisector), *hauteur* (height), *médiatrice* (perpendicular bisector), *bissectrice* (angle bisector). The word *médiatrice* is of Latin origin and thus does not 'show' what is meant by it. French textbooks often provide a mini-dictionary and a summary of definitions of geometrical terms at the back of the textbooks, so that the students can refer to it and look up terms. The English equivalent 'perpendicular bisector', or German equivalent *Mittelsenkrechte* says exactly what it is: it stands perpendicular to the side and it divides the side in two. Thus, when comparing the use of language in a geometry lesson, it appears that parts of the curriculum are easier learnt in one language than in another. In this case, geometry appears easier in German and English than in French. Geometry is highly regarded in France, in particular Euclidean geometry. It is believed that with the help of Euclidean geometry logical thinking and reasoning can be enhanced.

Secondly, this situation is characteristic for France, because there are clear competencies and mathematical notions that teachers have to teach in a certain year. These are defined by the curriculum- *les programmes*. Teachers know what should have been taught and learnt the previous year, and they refer to and build on it in lessons. Thirdly, French teachers typically ask pupils to keep two kinds of books: the *cahier de cours*; and the *cahier d'exercices*. In the *cahier de cours* pupils are expected to record the *cours* – the essence of the lesson, written in sentences together with a worked example. The *cahier d'exercices* is for exercises and any kind of preparatory work for the *cours*. This routine can be understood from traditions to keep 30 children together. Teachers commented that with the *cahier de cours,* those pupils who did not understand during the lesson had the chance to learn at home with the help of what was recorded in the *cahier de cours*. French teachers were trying to teach pupils as a whole class and, although they were aware that not everybody might have understood at the end of the lesson, they knew that at least pupils recorded the main points of the lesson in their *cahier de cours*. Pupils might have got to different stages in the lesson, but at the end of it, everybody was writing the statement and an example in their *cahier de cours*, so that they could learn and revise the lesson at home if necessary.

This has to be viewed in the French context of cultural and educational traditions. In terms of philosophical underpinnings, France is one of the heartlands of encyclopaedism, with its associated principles of rationality and *égalité*. The principle of rationality encourages the teaching of subjects which are perceived to encourage the development of rational faculties. Mathematics counts as one of those subjects, and it thus has a high status in France. Egalitarian views aspire to remove social inequalities through education and promote equal opportunities for all pupils. Every pupil has the *right* to be taught the entire curriculum, and in ways that are thought to benefit the majority of pupils. This is reflected in, for example, their use of the same way of division (see later), and routines such as the *cahier de cours*.

In contrast, the main underpinning philosophy of the English education system is humanism, with its associated principles of individualism, amongst others. English education is said to be child-centred and individualistic, and the interaction between teacher and pupil is greatly emphasised. One of the claims about humanism is that it is anti-rational and that England has in the past given 'little weight in education to rational, methodical and systematic knowledge objectives' (Holmes and McLean 1989). This can be understood in the light of the philosophy of humanism which assumes that to acquire knowledge is not a logical, sequential and standardised process, as rationalists would claim, but learning is regarded as 'intuitive'. The acquisition of knowledge was the outcome of the interaction between the inherent qualities of the learner and different materials appropriate to the student's development. Therefore, the content of education should be selected in the light of individual differences.

Germany espouses mainly humanistic views, based on Humboldt's ideal of humanism. Humboldt's concept of *Bildung* searches for 'rational understanding' of the order of the natural world. It incorporates encyclopaedic rationalism as well as humanist moralism, and basically promotes the unity of academic knowledge and moral education. The German humanist rationale is never allowed to avoid the importance of the study of mathematics and science subjects. Examples of the two latter cultural traditions (England and Germany) are given under 'genres' and 'the language of texts'.

Genres

> "If a reader is to be satisfied with a piece of sustained prose, whether it be a story, an account of a scientific experiment, a record of events, or just a paragraph of instructions as to how to get to one place from another, there has to be some 'shape' to it. In other words, there will be an organisational pattern evident within the writing."
>
> (Gannon 1985, p.57)

In all language discourse there are structural forms which are used to make meaning and which are accepted by the social system. The social context in which mathematical texts are generated carry with them the underpinning beliefs and philosophies that give life to the structure and definitions of the text. *Genres* are, according to Mousley and Marks (1991), conventionalised forms of texts. They refer to the ways we use language, and we use traditional patterns of language for specific purposes. Different genres have different structures and intentions. The choice of different genres is prescribed by individual situations, needs and purposes within the traditions of a culture.

> "Discourse carries meanings about the nature of the institution from which it derives; genres carry meanings about the conventional social occasions on which texts arise." (p.20)

Examples of genres are structures of mathematics textbooks in England, France and Germany (Haggarty and Pepin, 2002). For example, French mathematics textbooks are structured in a very particular way. Firstly, they are usually divided into three sections according to the structure of the *programmes* (the curriculum): numbers and algebra; statistics; and geometry. Every chapter is then divided into three parts: *activités; l'essentiel; exercices* (activities- essential- exercises). The activities are small investigations, practical or cognitive activities (sometimes bordering on exercises) which are intended to introduce pupils to a notion. Teachers usually choose one or several of those activities. *L'essentiel* corresponds to the essential part that needs to be taught and understood, in words and in worked examples. This is the *cours* and teachers usually compose their own *cours* for pupils to write in their books. The third part accommodates exercises, sometimes in order of difficulty.

The part that distinguishes French from English and German textbooks is, amongst other factors, the *activités* (small investigations) part. In German textbooks, after a short section with selected introductory exercises and the main 'message' or formula followed by worked examples, the majority of the sections consists of exercises. English textbooks also offer mainly exercises, interspersed with some points for discussion or investigations.

Why is it different in France? There is clearly an understanding in France that these cognitive activities help pupils to understand the notion being introduced by the teacher. In contrast to the 'old' *cours magistral* (lecture type teaching), teachers and inspectors claim that the activity approach is a 'softer' way to teach mathematics. In terms of French educational traditions, it seems to fit in with Piaget's notions of constructivism and their associated teaching approaches. In the French pedagogy, teachers focused on developing mathematical thinking which included exploring, developing and understanding concepts, and mathematical reasoning. They tried to forge links between skills and cognitive activities on the one hand, and concepts on the other. Relatively little time was spent on routine procedures. The emphasis was on process and not the result. These approaches reflect the ideal of rationality (in encyclopaedism) embodied in the notion of *formation d'esprit*.

On the other hand, in English classrooms the major aim was to convey a mathematical concept and let pupils get as much practice as possible, with the help of exercises from the textbook. The emphasis was on the skill side of mathematics and results- all approaches that can be traced back to (English) humanistic philosophies which do not emphasise the rational training of the mind.

In Germany, teachers' pedagogies reflected a relatively formal view of mathematics which included logic and proof. The teacher's role was that of the explainer who taught the structure of mathematics through an 'exciting' delivery and by adapting the structured textbook approach meaningfully. Logical thinking, the core of German humanist tradition, was regarded as important. The invention of new solutions or procedures was not encouraged, and lessons appeared relatively formal and traditional in terms of their mathematical content.

The language of texts

> "[The way we usually think of 'meaning' is] conditioned by centuries of written language. We are inclined to think of the meaning of words in a text rather than of the meaning a speaker intends when he or she is uttering linguistic sounds. Written language and printed texts have a physical persistence. They lie on our desks or can be taken from shelves, they can be handled and read. When we understand what we read, we gain the impression that we have 'grasped' the meaning of the printed words, and we come to believe that this meaning was in the words and that we extracted it like kernels out of their shells. We may even say that a particular meaning is the 'content' of a word or of a text. This notion of words as containers in which the writer or speaker 'conveys' meaning to readers or listeners is extraordinarily strong and seems so natural that we are reluctant to question it."
>
> (von Glasersfeld 1983, p.51-52)

In this part, we focus on textbooks as part of the written and spoken mathematical language. In English, French and German society, written texts hold a powerful place. This is exemplified in the above statement by von Glasersfeld. Previous research (Pepin and Haggarty, 2001) has shown that textbooks are extensively used by teachers in the three countries, and that teachers' constructs of mathematics are manifested in their practices (Pepin, 1999a) which are, in turn, underpinned by the educational and cultural traditions of the individual countries (Pepin, 1999b). It is suggested that within a particular country, textbooks reflect the significant views of what mathematics is, the mathematics that students need to know, and the ways that mathematics can be taught and learnt. Thus, what appears in mathematics textbooks is influenced by the multi-faceted aspects of an educational culture, and can therefore provide a window onto the mathematics education 'world' of a particular country. It is also assumed that teachers mediate mathematics textbooks in their lessons in different ways. In France and Germany, for example, the textbook is regarded as the key element of teaching and learning, whereas in England textbooks are viewed as one of many resources that teachers use in their classroom.

In all three countries, to a greater or lesser extent, textbooks are used for three kinds of activities: for teaching in order to lay down rules and conditions; for explaining the logical processes and going through worked examples; and for the provision of exercises to practice. Teachers in all three countries emphasised the use of textbooks for exercises. There were, however, differences in the extent teachers used them with respect to the two other categories. French teachers, for example, used the books for explanations, but 'insisted' on providing the rules and essence of the lesson (*cours*) without and in a different way than the book. German teachers purposefully used different worked examples than those in the textbooks, in order to initiate class discussion about the problems that might be encountered.

In terms of pedagogy, and this supports teachers' use of textbooks, English teachers spent relatively little time on explaining concepts to the whole class. Unless the lesson took the form of an 'investigation', most English teachers introduced and explained a concept or skill to students, gave examples on the board and then expected pupils to practice on their own in small groups. They usually gave them exercises to do from the textbooks, while they saw it their duty to attend to individual pupils. This can be understood in the light of traditions of individualism, one of the humanistic ideals. There was the espoused view that teachers had to attend to the need of the individual child.

French, and in particular German, teachers devoted a substantial proportion of the school day to whole-class teaching. French teachers, reflecting egalitarian views, expected the whole class to move forward together. They tried to pose thought-provoking problems, or chose cognitive activities from the textbook, and expected students to struggle with them. Then they drew together ideas from the class and the whole class discussed solutions which usually led to the formulation of the *cours*. German teachers used a more conversational style where they tried to involve the whole class in a discussion about a particular problem. The emphasis was on understanding, part of Humboldt's humanistic ideals. Typically, a teacher brought pupils to the board and discussed their mistakes and understanding with the whole class.

Teachers often assume that if books (and tests) are written, and in some countries selected by the ministry, for a specific grade level, most students of that age or grade will be able to understand the material. Studies into the readability of mathematics texts, for instance, have been carried out (for example, Fitzgerald 1980), with the result that generally 'readability levels' are recognised to be too high for the intended readership. In Germany and England, different textbooks are published for different achievement groups. In Germany, differences are made between mathematics textbooks for the three school forms of the tri-partite system: the *Gymnasium* (grammar school); the *Realschule* (technical school); and the *Hauptschule* (secondary modern). In England, where pupils are usually either 'streamed' in achievement-oriented form groups, or ‘setted’ for each subject, three ‘levels’ of textbooks exist for different achievement groups which differ in content as well as in text complexity. Only in France is it expected that all pupils follow the same textbook in any particular year. This particularity of France can be viewed in the light of egalitarian views, but also in terms of historical developments which were, in turn, influenced by prevailing cultural and philosophical traditions. Historically, the Haby reforms of 1975 established an essentially common core of lower-secondary education, the *collège unique*, and in 1977 a common curriculum was introduced. Since then, the subsequent education ministers have fought hard to prevent *les filières* (streaming). They argue that every child has the right to the entire curriculum which is reflected in a common textbook for all pupils of an age group.

Written ways of representing mathematical calculations are also linked to common practices in individual countries. One example is the representation of division. In England, different teachers and primary schools prefer different ways of writing division. This has as a result that in year 7, English secondary teachers are faced with the problem of ‘harmonising’ 30 children’s ways of representing division, together with their associated structures of thinking. In Germany and France, this is more standardised. The following examples refer to Germany and France respectively.

Germany:
$$\begin{array}{l}
171 : 5 = 34{,}2 \\
\underline{15} \\
\;\;21 \\
\;\;\underline{20} \\
\;\;\;\;10 \\
\;\;\;\;\underline{10} \\
\;\;\;\;\;\;0
\end{array}$$

France:
$$\begin{array}{r|l}
171 & \underline{5\quad\;} \\
21 & 34{,}2 \\
10 & \\
0 &
\end{array}$$

Whilst there is no explanation how these different representations have developed, it is nevertheless significant that German and French children are obliged to use one way of representing division, whereas English children are encouraged to use what they feel helps their knowledge construction best. One could argue that individualism in England supports this attitude, whereas egalitarian views in Germany and in particular in France necessitate that all children need the same way of calculating and representing long division.

Conclusions

> “Every child in every society has to learn from adults the meaning given to life by his society; but every society possesses with a greater or lesser degree of difference, meanings to be learned. In short, every society has a culture to be learned though cultures are different.”
>
> (Levitas 1974, p.3)

Intentionally or unintentionally, teachers mediate and teach the language of mathematics in their classrooms, and pupils are given the opportunity to speak and write mathematics. On the one hand, there is the particular culture of the mathematics classroom which Nickson (1992) describes as

> "the product of what the teacher and pupils bring to it in terms of knowledge, beliefs, and values, and how these affect the social interactions within that context. It is all too easy to assume that these invisibles of the cultural core are shared by all participants and that there is a harmony of views about the goals being pursued and the values related to them. … There is more possibility for choice and more possibility that those choices will be guided by different beliefs and values. Consequently, there will be greater variation in the cultures of mathematics classrooms."
>
> (Nickson 1992, p.111)

The particular context of the classroom is also part of the larger institutional (school) and societal context with its embedded values, beliefs and traditions of a particular education system which may be manifested in adopted curricula, educational practices, systemic features, to name but a few. These institutional and societal features represent a second source of influence on the language in mathematics classrooms.

This paper has attended to language in mathematics classrooms in three European countries: England, France and Germany. It is concerned with the ways the particular culture of the mathematics classroom and the culture at large influence language and communication.

Every country or system has its own language 'rules' which are underpinned by the system's cultural and philosophical traditions, and it is through language forms that these are mediated. From teachers' discourse and the written texts in textbooks, pupils receive powerful messages about the nature of mathematics, about its teaching and learning and about its value in society. Teachers teach their pupils, albeit unconsciously, which forms of knowledge and communication merit recognition and are acceptable within the dominant culture and traditions of any society, and hence within the classroom. It is important to recognise that language in mathematics carries meanings which are influenced by a complex mixture of teachers and pupils' personal conceptions of mathematics teaching and learning, and each country's educational and intellectual traditions. Thus, it is suggested that a more language-sensitive approaches to the teaching of mathematics is to be encouraged. Furthermore, it is argued that language in mathematics education needs to be understood in terms of the larger cultural context, and that an understanding of the wider meaning of language forms in a particular context can enhance communication between students and teachers, as well as between those involved in mathematics education across countries.

References

Bishop, A.J. (1992) 'Cultural Issues in the Intended, Implemented and Attained Mathematics Curriculum', in G.C. Leder (ed) *Assessment and Learning of Mathematics*, Hawthorn, Victoria: Australian Council for Educational Research, pp.169-89.

Fitzgerald, G.G. (1980) 'Readability of the Fry sample procedure', *Reading Research Quarterly* 15 (4): 489-93.

Gannon, P. (1985) *Assessing writing: Principles and Practice of Marking Written English*, London: Edward Arnold.

Haggarty, L. and Pepin, B. (2002) 'An investigation of mathematics textbooks and their use in English, French and German Classrooms: who gets an opportunity to learn what?' , *British Educational Research Journal* 28 (4).

Halliday, M.A.K. (1978) *Language as Social Semiotic: The Social Interpretation of Language and Meaning*, London: Edward Arnold.

Halliday, M.A.K. and Hasan, R. (1985) *Language, context and text: Aspects of language in a social semiotic perspective*, Geelong, Victoria: Deakin University.

Harris, P. (1987) *Measurement in Tribal Aboriginal Communities*. Darwin: Northern Territory Department of Education.

Harris, S. (1990) *Two way aboriginal schooling.* Canberra: Aboriginal Studies Press.

Holmes, B. and McLean, M. (1989) *The Curriculum- a comparative perspective*, London: Routledge.

Lean, G.A. (1992) *Counting systems of Papua New Guinea and Oceania.* Unpublished PhD thesis, Papua New Guinea University of Technology, Lae, Papua New Guinea.

Lean, G.A. (1995) *Counting systems of Papua New Guinea and Oceania* (3rd ed., Vols. 1-24). Lae, Papua New Guinea: Papua New Guinea University of Technology.

Legrand, P. (1997) *Les maths en collège et en lycée*, Paris: Hachette.

Levitas, M. (1974) *Marxist perspective in the sociology of education*. London: Routledge & Kegan Paul.

Mousley, J. and Marks, G. (1991) *Discourses in Mathematics*, Geelong, Victoria: Deakin University.

Pepin, B. (1999a) Epistemologies, beliefs and conceptions of mathematics teaching and learning: the theory, and what is manifested in mathematics teachers' practices in England, France and Germany. In Hudson, B., Buchberger, F., Kansanen, P. and Seel, H. (1999) *Didaktik/Fachdidaktik as science(s) of the teaching profession. TNTEE Publications*, Volume 2 (1), pp. 127-46.

Pepin, B. (1999b) The influence of national cultural traditions on pedagogy: classroom practices in England, France and Germany, in J. Leach and B. Moon (eds) *Learners and Pedagogy*, London: Sage Publications.

Pepin, B. and Haggarty, L. (2001) 'Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures', *Zentralblatt for the Didactics of Mathematics*, 33 (5).

Pepin, B. (2002) ‘Different cultures, different meanings, different teaching’ in L. Haggarty (ed) *Teaching Mathematics in Secondary Schools*, London: Routledge.

Nickson, M. (1993) The culture of the mathematics classroom: an unknown quantity? in D.A. Grouws (ed) *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillian Publishing Company.

von Glasersfeld, E. (1983) ‘Learning as a constructivist activity’, in J.C. Bergeron and N. Herscovics (eds) *Proceedings of the Fifth Annual Meeting*, vol.1, Montreal: The North American Chapter of the International Group for the Psychology of Mathematics Education, pp.41-69.

Zevenbergen, R. (1995) *The construction of social difference in mathematics education*. Unpublished PhD thesis, Deakin University.

Language across the mathematics curriculum: some aspects related to cognition

Florence Mihaela Singer, Institute for Educational Sciences, Romania

Abstract

Contemporary society asks for a broad range of competencies that deal with communication and meta-cognition. In this context, there is a need to train language skills in mathematics in a more systematic way than before. Recent research in cognitive psychology offers some support for this idea, revealing a number of similarities beyond the differences: language and mathematics both have computational properties that are specifically processed by the human mind; language and mathematics are both embodied in human cognition; language, as well as numerical abilities contains inborn components of the human propensities for learning. Consequently, it is likely to have a positive effect on learning by stressing language and mathematics interaction in teaching and by valuing their common properties. A key factor in achieving these goals is to develop a competence-based curriculum that highlights various types of transfer.

Introduction

If learning mathematics supposes merely acquiring techniques for computing and solving categories of well-classified problems in order to "train the mind", then there is no need to pay attention to language and communication skills in mathematics. Paper-and-pencil, basic symbols, and formulas are sufficient tools to show math performance. Indeed, these tools proved to be adequate for decades. Today, however, in a socio-economical variable environment, the problems people are confronted with on the job market, or even in everyday life, are ill-defined. Far from being typical, these problems do not allow, unfortunately, already known algorithmic solutions. The solutions become more fluid, they need explanations, argumentations, sometimes they need negotiated meanings, or additional parameters to partially uncover the underlying complexity of the real world. Strategies that not long ago were only for scientific and research use are now needed for ordinary people in everyday life problem solving. In this context, we might consider PISA's definition of *Mathematics literacy*: "An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen." This new context launched the issue of explicitly cultivating language and communication across the mathematics curriculum.

This article is focused on regarding language in mathematics education from a cognitive perspective. Recent research in cognitive science and neuroscience revealed two aspects that are contradictory. On the one hand, it seems that the number sense is active in preverbal children; on the other hand, although they show precocity of numerical ability at very young ages, later, most of students have difficulties in explaining their computing strategies or their approaches to problem solving. However, as we have discussed above, contemporary society asks for a broad range of competencies that deal with communication and meta-cognition. Therefore, there is a need to train language skills in mathematics starting with the earliest ages of the mathematics instruction. Below, we discuss how bridges to this target can be offered by the study of cognition.

Stressing the interaction in teaching between language and mathematics, profitable effect can be identified on learning. Nevertheless, we have to face the threat of a kind of subtle resistance on the part of mathematics teachers, who appreciate the "encrypted" characteristics of mathematics. Within a new paradigm, teachers are supposed to teach not only mathematics, but also how to communicate mathematically. Here, we have to take into account that without a specific training that is focused on transfer, the application of knowledge from one domain to another one is only accidentally done by the human mind. To enhance the transfer capabilities in school learning, it is necessary to develop a competence-based curriculum that highlights various types of transfer. Because mathematics is highly conceptual and structured, this change in approaching teaching and curriculum is significant and needs special preparation. These ideas will be detailed in what follows.

Designing a competence-based curriculum – a necessity of our times

The large mass of workers in factories in the first half of the twentieth century has been replaced nowadays by a large mass of people working in services within a global market. While the industrial worker required basic instruction to develop the ability to handle clear, specific driven tasks in a large driven mechanism, today's employee needs the ability to communicate, to think fluently, and to adapt him/herself to a variable environment, including coping with changing the profession many times in a lifetime. However, the schools have been ineffective in meeting these needs, as demonstrated by the fact that most of today's graduates show poor ability to transfer their skills from school to job, from one domain of knowledge to another, or from one job to a new one. While a "drill and practice" technique assured the success of the economy in the industrial era, this way of learning and approaching problems is largely inappropriate today. What roles does the knowledge society expect of schools? From a holistic perspective, these roles can be characterised by two words: *dispersion* and *extension* (Singer, 2007a). School as a knowledge institution needs to be more fluid (i.e. the borders between formal, informal and non-formal learning tend to be diffuse) and more extensive (i.e. the duration of school learning extends beyond the ages it used to cover, giving rise to concepts such as lifelong learning). In Drucker's words, "access to the acquisition of knowledge will no longer be dependent on obtaining a prescribed education at any given age. Learning will become the tool of the individual available to him or her at any age if only because so much of skill and knowledge can be acquired by means of the new learning technologies" (Drucker, 1994, p. 4).

Research in cognitive science stresses that learning within a domain is efficient (maximum educational benefits with minimum effort and resources) and effective (meaningful and relevant for real-life problem solving) if it is focused on the acquisition of the domain's specific symbol system and procedures, i.e. the very entities that enable the structuring and functioning of a specific thinking mode that allows adequately processing new contexts (e.g. Bransford et al, 2000; Gardner, 1983, 1991; Singer, 2003b). More specific, the successful learners are the ones who are able to reorganise their already acquired structured sets of skills and knowledge in order to obtain new procedural configurations that are adequate to new situations and to new problem understanding and solving (Singer & Sarivan, 2006). This process is valid at all levels of instruction as well as for the knowledge progress in general.

Designing a competence-centred curriculum is in line with the results of research in the field of cognitive psychology, according to which competencies are the best means to transfer and use knowledge and skills in new and dynamic situations/contexts. A functional definition of competence might be: a structured set of knowledge and skills acquired through learning, which allows individuals to identify and solve, in a variety of contexts, problems that are characteristic for a certain domain of activity (Singer, 2003a, 2006a). A competence-centred model of curricular design simplifies the curriculum structure and ensures a higher efficiency of the teaching/learning and assessment processes. This allows for operating at all levels with the same unit: the competence, capable of orienting the actions of all the actors of the educational process: curriculum designers, assessment specialists, teachers, inspectors, students, parents. A competence-based curriculum can better answer the current needs of social and professional life, of the labour market, focusing teaching on the pupil's acquisitions.

Language and mathematics – distant relatives?

The following paragraphs give some insights into the relationships between language and mathematics from a cognitive perspective. As Ongstad has emphasised, mathematics is 'conceptual' to the extreme: students will face problems understanding mathematics as a language on its own (its conceptual framework). Therefore, there is a need to identify ways not only to make this language accessible, but language should also serve as a tool in facilitating access to understanding mathematical concepts.

Over the last three decades, a large body of research has been devoted to analysing infants' cognitive capacities. An important category of experimental findings related to the subjects of this study show that preverbal children grasp some aspects involving quantities. It seems that the number sense is active in infants before they are able to use language. Thus, Wynn (1990, 1992), and Starkey (1992) showed that 5-month-old infants are able to compare two sets of up to three objects and to react when the result of putting together or taking away one object is falsified. These experiments were followed by many replications and extensions. Using the infant's gaze patterns it was possible to show that babies as young as 5 months are able to identify differences in numbers of objects up to three (Canfield and Smith, 1996). Infants looked longer at arrays presenting the wrong number of objects, even when the shapes, colours, and spatial location of the objects in both displays were new (Simon et al., 1995; Koechlin et al., 1997). A series of experiments suggested that number representation in humans has at least three components (Dehaene, 1997; Dehaene et al., 1999; Spelke, 2003): one for recognising numerosity limited up to four items at a glance, without counting – subitizing (e.g. Benoit et al., 2004; Gallistel and Gelman, 1991; Mandler and Shebo, 1982; Starkey et al., 1990), one for approximate numerosities (Dehaene, 1997; Gallistel and Gelman, 1992), and the third for large exact numerosities, in which the natural language interferes (e.g. Gelman, 1990). This area of neuroscience is important from an educational perspective because it shows that, far from being "tabula rasa" at birth, children have predispositions that allow them later to construct mathematics knowledge.

On the other hand, however, language plays an important scaffolding role for developing mathematical ability. I stress below the "scaffolding" (Vygotsky, 1934/86) function of language. As Clark (1995) has argued, language augments the existing computational abilities by externalising and recombining the information used by pre-linguistic computations in several ways. Clark sees language as fulfilling a Vygotskian scaffolding function: "Much of the true power of language lies in its under - appreciated capacity to reshape computational spaces which confront intelligent agents." Almost the same idea, expressed from a sociological anthropological perspective, is central to Lacan's psycho-linguistic conception (1966/1977).

Both domains, language and mathematics, have at least two common characteristics – computational properties and redundancy. Chomsky (1980) defines the faculty of language in a narrow sense as being the abstract linguistic computational system (narrow syntax) that generates internal representations and maps them into the sensory-motor interface by the formal semantic system. While the internal architecture of language supports many debates, there is an agreement that a core property of the faculty of language in a narrow sense is recursion, attributed to narrow syntax; this takes a finite set of elements (words, sentences) and yields an array of discrete expressions (Hauser, Chomsky and Finch, 2002), which can be considered potentially infinite. Similarly, from a set of a few digits, infinitely many natural numbers are generated through a recursive procedure given, essentially, by the Peano's axioms. The role of recursion is essential for mathematics, and, as traditionally Chomsky emphasised it, for language.

The embodied metaphors theory (Lakoff, 1987; Lakoff and Johnson, 1980) extends the syntax properties to the human conceptual systems. For Lakoff and his colleague, language is embodied, which means that its structure reflects our bodily experience, and our bodily experience creates concepts that are then abstracted into syntactic categories. They concluded that grammar is shared (to some degree) by all humans for the simple reason that we all share roughly the same bodily experience. Moreover, the core of our conceptual systems is directly grounded in perception, body movement, and experience, which integrate both the physical and social context. Going further on this line of research, *recursion* appears to be a general property, not of language, but of human thinking and through this, implicitly of language. Moreover, Lakoff and Nuñez (2000) explain that the structure of mathematics is built from various metaphors, ultimately grounded in our embodied reality. The metaphors are cognitive descriptions that express the way we think and understand; mathematics is a construct that makes use of metaphors (usually implicitly). More specific, three types of metaphors might be emphasised: *grounding metaphors* – metaphors that ground our understanding of mathematical ideas in terms of everyday experience, *redefinitional metaphors* – metaphors that impose a technical understanding replacing ordinary concepts, and *linking metaphors* - metaphors within mathematics itself that allow us to conceptualise one mathematical domain in terms of another mathematical domain.

For example, to explain how human beings construct the concept-process of infiniteness, Lakoff and Núñez put forward the hypothesis that mathematicians' ideas about infinity are originated by a single general conceptual metaphor in which processes that go on indefinitely are conceptualised as imperfective processes (the general name given by linguists to processes without an end). This metaphor is called the Basic Metaphor of Infinity (BMI) and its effect is to add a metaphorical completion to the ongoing process so that it is seen as having a result. Lakoff and Núñez argue that human beings conceptualise indefinitely continuous motion as repeated motion: "continuous walking requires repeatedly taking steps; continuous swimming requires repeatedly moving the arms and legs; continuous flying by a bird requires repeatedly flapping the wings. This conflation of continuous action and repeated actions gives rise to the metaphor by which continuous actions are conceptualised in terms of repeated actions." (Lakoff and Núñez, 2000, p. 157). They concluded that infinite continuous processes are conceptualised via this metaphor as if they were infinite iterative processes. What is to emphasize is the claim that metaphor does not reside in linguistic expressions alone, but also in conceptual structure. This is another reason for which domain-specific linguistic training (the mathematics language) should be ingrained in learning mathematical concepts and procedures.

The connection between language and mathematics is also highlighted by the development of mental operations. The child develops arithmetical operations that evolve from perceiving variation in quantity (within the so called proto-quantitative abilities) to mastering computing through the study in school of binary operations, such as addition and multiplication and their opposites: subtraction and division. As progressing in school learning, the algebraic operations increase in abstractness in two ways: by increasing the complexity of the numbers the operations apply to, and by the passage from objects to numbers and to symbolic expressions.

The category of logical operations extends the use of basic operators (conjunction, disjunction, negation) to the capacity of formulating logical inferences. Here the language has a decisive role, being essential for different types of reasoning. For example, different patterns are involved in deductive reasoning and non-deductive reasoning. In addition, different patterns are involved in different types of deductive reasoning, which can be: conditional, consecutive, causal, modal, normative, procedural. We can also identify two types of non-deductive reasoning: inductive and analogical. The daily reasoning and argumentation actually mix together many of these logical categories. In addition, many of our decisions are based on judgments that are not necessarily expressed in specific words. Yet, this description becomes necessary when analysing the mechanisms that underlie understanding, in order to develop appropriate training.

Mathematics is based on a variety of conventions (math symbols, math notational systems that have evolved through the centuries, conventional approaches in problem solving, etc.). Problem solving strategies were differently expressed along the history of mathematics, and the way in which a solution is understood and accepted is socially and historically determined. These conventions need to be learned explicitly because, as many studies have demonstrated, they are not necessarily internalised by learning mathematics concepts, they need separate training. For instance, learning how to solve a problem is not enough to know how to explain the solving in such a way to be understood by somebody else. Consequently, solving the problem and explaining the solution are different aspects and they both need to be trained explicitly and this training involves various aspects of communication.

Learning mathematics – building structural representations mediated by language

Mathematics deals with representations. In order to bring representational change to schools as an intrinsic phenomenon of learning, it is necessary to develop structural models that build relevant connections within the domain of study, and to make them part of the teaching-learning design. An adequate training based on these models may activate dynamic mental structures in students (Singer, 2007b). The key to an effective learning seems to be to help children building dynamic mental structures that can self-develop and generalise across new tasks in adequate contexts (Singer, 2001). Teaching should focus, on the one hand, to internalising a variety of representations and, on the other hand, to building ways to move from one representation to another one. Moreover, an appropriate training can help to move the connection language-mathematics to automatised procedures. Automaticity refers to the way we perform some mental tasks quickly and effortlessly, with little conscious thought or conscious intention. Automatic processes are contrasted with deliberate, attention-demanding, conscious, controlled aspects of cognition (Palmeri, 2001). Automatic processes seem to occur reflexively, while controlled processes require conscious intention to become initiated. This might optimise learning, because automatic processes are free from dual-task interference, i.e. they are not influenced by other tasks that are executed concurrently.

Traditionally, when learning mathematics, pupils practice only restricted areas of operations, usually the ones looking to be strongly related to the specific content (Singer & Voica, 2004). The results of this practice reflect an inconsistency in dealing with the basic concepts of the discipline and a huge difficulty in making connections and transfers. To overcome this situation, the teaching of mathematics should offer students opportunities to:

1. master and correctly use mathematical notation and terminology, in various contexts.
2. prove confidence and initiative in handling mathematical topics, in describing them, orally or in writing, and in supporting own work and the results obtained by means of intuitive arguments.
3. use mathematical ideas, rules and models, in tasking practical problems and everyday situations; understand the advantages offered by mathematics in tackling, clarifying, and tracking such problems or situations.
4. devise and solve exercises and problems; use standard methods, or adapt a known method, or imagine new solving paths, for this purpose.
5. compare and criticise different solutions of an exercise or problem, with respect to correctness, simplicity, and the significance of the results obtained.
6. engage in critical discussions concerning a mathematical subject, with peers or/with the teacher; state questions in order to clarify own ideas.
7. describe and compare concrete and mathematical objects; establish similarities and differences; select and classify such objects.
8. generalise and particularise ideas and methods.

More specifically, the competences that involve processing language in mathematics learning might be stimulated by practising the following categories of learning tasks, especially in compulsory schooling:

- Represent various types of numbers, variables and functions using different modalities. Translate from one representation to another.
- Compare different representations by emphasising correspondences among them. Use conventional symbols and terms.
- Express properties of mathematical operations (commutativity, associativity, neutral elements, and reversibility), by manipulating various representations. Use these properties for mental computing.
- Perform measurements using non-standard measures (such as pieces of plastic, or cardboard of different shapes and sizes). Chose the appropriate units for measuring a given object. Measure the same object using several measures (of different shapes or sizes). Record the

results and discuss about them. Recognise the need to use standard units in order to be able to compare dimensions of objects.

- Estimate the results of certain measurements, based on familiar measures or units; verify them by measuring.
- State correctly the relative positions of objects, drawings, or geometrical entities using appropriate terms.
- Sort out objects, drawings, or mathematical entities using given criteria. Discover and identify criteria suitable for classifying given objects/ mathematical entities.
- Complete sequences of shapes or objects that hide different patterns; find patterns; create patterns and make up the corresponding sequences; describe various patterns.
- Use various symbols to represent the unknown term in an exercise. Solve exercises containing such symbols.
- Recognise concrete situations or expressions of the common language that can be modelled by mathematical operations; use these expressions currently.
- Transform word problems into exercises and vice-versa: device a variety of problems that might be processed by solving a given exercise. Create various word problems, starting from a given exercise or from another explicitly stated requirement.
- Identify the elements of a word problem, or of a problem-situation (given data, unknown data, relations among data). Discuss about them before engaging in solving the problem.
- Reformulate given problems and/or construct variants within given conditions or without restrictions (e.g. change the text and maintain the data, change parts of the text, change the question, etc.).
- Contrast and critique solutions and approaches to the same problem; discuss the correctness and significance of the results.

The teacher as a double expert

The knowledge society displays a mass need for quality education. This requires large numbers of ”expert teachers” i.e. professionals who are able to find effective solutions to a wide range of instructional problems (Singer & Sarivan, 2006). The post-industrial era expects better trained teachers to better train students for new complex social demands. The ”expert teachers” perform a number of competences. Firstly, they should exhibit good mathematics competences. These are acquisitions that reflect the specific cognitive and attitude profile of the professional representing the mathematics field of knowledge.

To give a more concrete flavour of this idea, below there is a list of proposed competencies of the mathematics graduate, which are correlated with the 6th level of the European Qualifications Framework (Commission of the European Communities, 2005, 2006). According to Singer & Sarivan (2006), the graduate of mathematics at the university level should possess the following competences:

1. Identify relevant data, mathematical concepts and their relationships in order to solve practical/ theoretical problems.
2. Make use of algorithms in a variety of problem-solving situations or for the local/ global mathematical description of a concrete situation.
3. Make use of a specific symbol system in order to express quantitative and qualitative mathematical features and their processing algorithms for further study and communication to specialised/ non specialised audiences.
4. Generalise properties or algorithms in order to optimise problem solving strategies both individually and within a team.
5. Model a variety of situations and apply these models in real-life problem solving or in the design of various projects.
6. Show interest to identify patterns and to develop models and representations of the real world, including evaluation of personal approach to learning.
7. Develop hypotheses and assess their validity for an adequate management of study/work situations in which a variety of factors interact.
8. Use logical arguments to refer to ethical problems or social tensions in study/work contexts.
9. Develop a dynamic vision of Mathematics as a domain which is closely related to society by its role in the development of science, technology and social analysis.

How this new profile can be reflected in the practice of mathematics teaching? Prospective mathematics teachers ought to gain a mathematics thinking profile in order to facilitate competence transfer in their classes. Beyond the specific knowledge of what they are supposed to teach, a necessary condition is that teachers envisage mathematics as a process within its dynamics. The domain searches and fulfilments along history, the way in which the concepts have been clarified, enriched and extended are fundamental for understanding mathematics as a product of human culture and human mind. This understanding should be appropriately transmitted to pupils in secondary school, as well as in primary school.

In order to achieve this goal, prospective teachers should learn teaching competences that are specific to mathematics. Learning mathematics, they develop a specific thinking profile, which has some domain-specific attributes. These attributes should be valued by the preparation for the teaching profession. Thus, all teachers share a number of fundamental operational tools: planning, organising, assessing and reflecting on classroom activities. To make these tools efficient and well internalised, pre-service teacher training programs should create domain-specific contexts in which prospective teachers perform them within their field of expertise. Sarivan &Singer (2006) propose the following list of competencies for the mathematics teacher:

- Design a variety of activities in order to structure learning tasks that lead to identifying and overcoming difficulties in learning mathematics

- Perform motivating activities that address specific students needs in order to optimize learning
- Make use of an objective and transparent assessment oriented by the purpose to improve students' results
- Check the efficiency of the used methodology in order to improve the didactical activity
- Participate competently in the decision-making process at the level of the school as a learning organisation.

A mathematician develops her curriculum design competencies by constructing a structured didactical approach (N.B. structure is a key-concept in mathematics), which allows differentiating the methods depending on pupils' level of knowledge and understanding. The student in Mathematics structures a logical thinking mode that is based on assertions the truth-value of which can be precisely determined. The math teacher may efficiently transfer this acquisition in developing assessment items that ensure an objective appraisal of pupils and stimulate their progress in learning. In this process, communication is „mathematically oriented". Approaching multiple ways in problem solving and checking the correctness of the solutions are capacities the mathematicians develop when confronting their domain, and these can be seen from a communicational perspective. Such abilities are useful tools in reflecting about their own didactical approach in teaching. These also open the way in using multi-perspective in analysing and interpreting a variety of mathematical texts and in transferring this capacity to students. While developing acquisitions in the sphere of social relationships (teacher's role, students' roles, and classroom organisation), math prospective teachers learn to optimise strategies that they can use in organizing an engaging context for students. Moreover, from a social view, in a learning organisation, mathematicians can use their cognitive abilities in order to orient the school community toward pertinent decisions. The mathematics teachers can value their communication skills within the domain specific thinking profile and this interaction is likely to strengthen their own learning and their capacities to generate mathematics understanding in students.

Traditional mathematics teaching rarely stressed the value of communication understood in a broad sense. Consequently, innovative approaches should highlight both symbolic math communication, as well as verbal and social sides of communication. The first threaten further studies in mathematics, the second opens the doors of academic mathematics to practical applications. In conclusion, nowadays mathematics teachers cannot marginalise either of them.

References

Benoit, L., Lehalle, H. and Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development, 19*, 291–307.

Bransford, J.D., Brown, A.L. and Cocking, R.R. (Eds.). (2000). *How people learn: Brain, mind, experience and school*. Washington, D.C.: National Academy Press.

Canfield, R.I., Smith, E.G. (1996). Number-based expectations and sequential enumeration by 5-month-old infants. *Developmental Psychology*, *32*, 269-279.

Chomsky, N. (1980). *Rules and Representations*. New York: Columbia University Press.

Clark, A. (1998). Magic Words: How Language Augments Human Computation. In P. Carruthers and J. Boucher (Eds.) *Language and Thought: Interdisciplinary Themes*, Cambridge University Press: Cambridge, p. 162-183

Commission of the European Communities (2005). Commission staff working document, Brussels. *Towards a European Qualifications Framework for Lifelong Learning* http://www.bologna-edinburgh2004.org.uk/library.asp

Commission of the European Communities (2006). Implementing the Community Lisbon Programme, *Proposal for a Recommendation of the European Parliament and of the Council on the establishment of the European Qualifications Framework for Lifelong learning*, Brussels, 5.9.2006.

Dehaene, S, Spelke, E., Pinel, P., Stanescu, R., and Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*, *284*, 970-974.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford U.P.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20* (3/4/5/6), 487–506.

Drucker P. F. (1994). *Knowledge work and knowledge society. The social transformations of this century* The 1994 Edwin L. Godkin Lecture, available at www.ksg.harvard.edu/ifactory/ksgpress/www/ksg_news/transcripts/drucklec.htm

Gallistel, C., and R. Gelman. (1992). Preverbal and verbal counting and computation. Special issues: Numerical cognition. *Cognition*, *44*, 43-74.

Gallistel, C.R., and Gelman, R. (1991). Subitizing: The preverbal counting process. In F Craik, W. Kessen and A. Ortony (Eds.), *Essays in honor of George Mandler* (pp. 65-81). Hillsdale, NJ: Erlb. Assoc.

Gardner, H. (1983/1993). *Frames of mind: The theory of multiple intelligences*. New York: Basic books.

Gardner, H. (1991). *The Unschooled Mind*. New York: Basic Books

Gelman, R. (1990). First principles organize attention to relevant data and the acquisition of numerical and causal concepts. *Cognitive Science*, *14*, 79-106.

Hauser, M.D., Chomsky, N., Fitch, W.T. (2002). The faculty of language: What is it, who has it, and how did it evolve? *Science*, 298, 1569-1579.

Hirschfeld, L.A. şi Gelman, S.A. (eds.) (1994). *Mapping the mind: Domain specificity in cognition and culture*, New York: Cambridge University Press.

Koechlin, E., Dehaene, S., and Mehler, J. (1997). Numerical transformations in five month old human infants. *Mathematical cognition*, *3*, 89-104.

Lacan, J. (1966/1977). *Écrits*, Paris: Seuil, 1966. Écrits, A Selection, New York: Norton, 1977.

Lakoff, G. (1987). *Women, fire and dangerous things: what categories reveal about the mind*, Chicago: University of Chicago Press.

Lakoff, G. and Nuñez, R. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. New York: Basic Books.

Lakoff, G., & Johnson, M. (1980). *Metaphors We Live By.* Chicago: University of Chicago Press.

Mandler, G. & Shebo, B.J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, *11*, 1-22.

Ongstad, S. (ed) (2007). Language in Mathematics? A comparative study of four national curricula, Council of Europe

Palmeri, T.J. (2002). Automaticity. In L. Nadel et al. (Eds.), *Encyclopedia Of Cognitive Science* (488). London: Nature Publishing Group.

Sarivan, L., Singer, F. M. (2006). A Competence-based Curriculum for Initial Teacher Training: From Good Intentions to the Real Fact in F. M. Singer and L. Sarivan (Eds.) *QUO VADIS ACADEMIA? Reference Points for a Comprehensive Reform of Higher Education* (In Romanian, with a synthesis in English), Bucharest: Sigma, pp.236-287.

Simon, T., Hespos, S., and Rochat, P. (1995). Do infants understand simple arithmetic? A replication of Wynn (1992). *Cognitive Development*, *10(2)*, 253-269.

Singer, F. M., (2001): Structuring the information – a new way of perceiving the content of learning, *Zentralblatt für Didaktik der Mathematik/IRME, MATHDI*, 6, 204-217.

Singer, F. M. (2003a). Designing a Competence Based Curriculum – Taking on The Challenge (In Romanian), *The Pedagogical Review*, p. 69-84.

Singer, F. M. (2003b). *From cognitive science to school practice: building the bridge.* In N.A. Pateman, B.J.Dougherty & J. Zilliox (Eds.), *Proceedings of the 27th PME.* Hawaii: CRDG. 4, 207-214.

Singer, F. M. (2006a). A Cognitive Model for Developing a Competence-based Curriculum in Secondary Education. In: Al. Crisan (Ed.), *Current and Future Challenges in Curriculum Development: Policies, Practices and Networking for Change*. Bucharest: Education 2000+ Publishers.

Singer, F. M., Sarivan, L., (coord., 2006). *QUO VADIS ACADEMIA? Reference Points for a Comprehensive Reform of Higher Education* (In Romanian, with a synthesis in English), Bucharest: Sigma.

Singer, F. M. (2007a). Balancing Globalisation and Local Identity in the reform of Education in Romania. In B. Atweh, M. Borba, A. Barton, D. Clark, N. Gough, C. Keitel, C. Vistro-Yu, and R. Vithal (Eds), *Internalisation and Globalisation in Mathematics and Science Education*, New York: Springer Science, Chapter 20, pp. 365-382.

Singer, F. M. (2007b). Beyond Conceptual Change: Using Representations to Integrate Domain-Specific Structural Models in Learning Mathematics. *International Mind, Brain, and Education Journal*, 1 (2), 84-97, Boston: Blackwell Publishing.

Singer, F. M., Voica, C. (2004). *Challenging the future: mathematics education in Romania between ideals and reality*, Cub, ICME-10.

Spelke, E. (2003). What Makes Us Smart? Core Knowledge and Natural Language. In D. Gentner & S. Goldin-Meadow (Eds.), *Language in Mind* (pp. 277-311). Cambridge, MA: MIT Press.

Starkey, P. (1992). The early development of numerical reasoning. *Cognition*, *43*, 93-126.

Starkey, P., Spelke, E., and Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, *36*, 97-127.

Vygotsky, L. S. (1934/1986). *Thought and Language.* Cambridge, MA: MIT Press.

Wynn, K, (1990). Children's understanding of counting. *Cognition*, *36*, 155-193.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*, 220–251.