LANGUAGE AND SCHOOL SUBJECTS

LINGUISTIC DIMENSIONS OF KNOWLEDGE BUILDING IN SCHOOL CURRICULA

N° 4

Items for a description of linguistic competence in the language of schooling necessary for learning/teaching <u>mathematics</u> (end of compulsory education)

An approach with reference points

This text has been produced for the *Platform of resources and references for plurilingual and intercultural education* by

Helmut Linneweber-Lammerskitten

Language Policy Unit
Education Department, DGII
Council of Europe, Strasbourg
www.coe.int/lang

LIST DOCUMENTS WHICH PROPOSE ELEMENTS FOR THE DESCRIPTION OF LINGUISTIC COMPETENCE FOR SPECIFIC SCHOOL SUBJECTS

1. *Items for a description of linguistic competence in the language of schooling necessary for learning/teaching history (end of obligatory education)*

   An approach with reference points - Jean-Claude Beacco

2. *Items for a description of linguistic competence in the language of schooling necessary for learning/teaching sciences (end of compulsory education)*

   An approach with reference points – Helmut Vollmer

3. *Items for a description of linguistic competence in the language of schooling necessary for learning/teaching literature (end of compulsory education)*

   An approach with reference points – Irene Pieper

4. ***Items for a description of linguistic competence in the language of schooling necessary for learning/teaching mathematics (end of compulsory education)***

   An approach with reference points – **Helmut Linneweber-Lammerskitten**

Items for a description of linguistic competence in the language of schooling necessary for teaching and learning mathematics in secondary education (end of compulsory education) - An approach with reference points

This text presents a procedure to help in creating a curriculum for the teaching and learning of mathematics which explicitly takes into account the discursive and linguistic dimensions of this subject area. It mainly transfers and adapts the ideas and procedures developed for history in Beacco (2010), science in Vollmer (2010) and literature in Pieper (2011). It proceeds through successive stages, for which there are corresponding inventories of references, from the level of educational goals in the teaching and learning of mathematics to the identification of linguistic elements which it is particularly important to systematise in the classroom in order to manage the corresponding forms of discourse.

The texts in this part of the platform – on history, sciences, literature and mathematics – all contribute to the identification of the linguistic dimensions of knowledge building in school curricula. They aim at offering assistance for coherent curriculum development and express shared values. For this reason, the texts provided follow a common pattern: some parts are nearly identical in wording, other parts have been adapted to the peculiarities of mathematics education.

TABLE OF CONTENTS

0. Introduction

Though it is widely accepted among mathematicians that mathematics itself can be seen as a universal language for all sciences, language awareness is often missing among mathematicians at universities and mathematics teachers at school. There is still a widespread prejudice that natural language and linguistic competences are less important, if not irrelevant, for the understanding and practicing of mathematics and that therefore language or even linguistic competence should not form part of a curriculum of the subject mathematics at school.

With the publication of the OECD/PISA concept of mathematical literacy and the development of educational standards and competency models in several European countries things begin to change: Though the term “literacy” in the PISA 2003 study is used only as a metaphor, it nevertheless imports some linguistic connotations and the concept of mathematical literacy itself goes far beyond operating and calculating with numbers. In the recent formulation of educational standards (in several European countries), which refer to competency models, linguistic aspects - especially those concerning argumentation and explanation - are seen as integrated parts of mathematical competence as a whole. Following this line of thinking, language awareness and the integration of linguistic aspects in teaching and learning mathematics should be part of any mathematics curriculum.

This paper presents

- an overall approach for the description and categorisation of the language competences needed for successful learning/teaching in mathematics education
- open-ended reference points (in the form of inventories/checklists) which are to be completed by users, according to the specifics of the respective educational system and the languages in which teaching is conducted.

The purpose of these reference points is to help users in:

- identifying the linguistic activities present in the subject under consideration;
- specifying the forms of the language of teaching/learning required in mastering the varieties of discursive content attached to the subject and the forms of communication necessary for imparting and acquiring subject-related knowledge and skills.

The overall scheme of the approach is as follows:
(1) inventory and description of the educational values targeted by mathematics teaching practices;
(2) inventory and description of the social situations of communication involving mathematics in the learners’ social environment;
(3) inventory and description of some basic /the expected mathematical knowledge structures;
(4) inventory and description of the existing in-school communication situations for the acquisition and construction of basic knowledge and procedures in mathematics.

Based on steps (1) to (4) it is then possible to create:
(5) inventories and descriptions of the specific linguistic, discursive and semiotic characteristics of relevance for the types of discourse involved in mathematics teaching and learning practices; these characteristics deserve to be taught in their own right in this subject area.

In other words, what is proposed here is a common procedure, whatever the language of instruction in question is, whether it be the learners’ first language or an additional language acquired to a standard of proficiency of at least level B2, according to the CEFR (*Common European Framework of Reference for Languages* – Council of Europe 2001).

1. Educational values and mathematics education (inventory and description of the educational values targeted by mathematics teaching practices)

All teaching pursues educational goals over and above the expertise and learning which are both its substance and its aspiration. The overall objectives and values of education – and hence of mathematics teaching and learning - are twofold: they concern the personal welfare of the individual and the public welfare of society, as well as rights and duties on both sides. Learners are entitled to acquire certain competences, skills, knowledge and experiences as prerequisites for a successful future life in different respects, such as:

for developing their own identity (personal domain)
for participation in society as social agents and democratic citizens (public domain)
for finding their place on the job market (occupational domain)
for their future learning[1] (educational domain)
…

Society in turn puts requirements on learners: they are expected to use the opportunities for learning offered to them and to take efforts to acquire the necessary competences for their future, especially for their future role as democratic citizens. The values of individual and public welfare and the rights and duties combined with these constitute a basis of legitimation for education and educational objectives and goals.

Mathematics education is expected to contribute to these objectives and can do so in many different ways and on different levels. On a general and abstract level it can help students to acquire:

mathematical competence in the sense of mathematical literacy (OECD 2003), which presupposes numerous subordinated competencies, as well as skills, declarative and procedural knowledge, abilities, emotions, volitions and so on
key competences which are not specifically mathematical in character[2], but can be supported or frustrated by mathematical education
mathematical experiences that can probably only be made in an artificial environment of a mathematics classroom
…

The role of languages of education in schools is to structure and assist the training and education of social actors and the development of the individual to their full potential as individuals. The aims of this training/education are shared by the Member States of the Council of Europe as the basis for living in society in Europe.

1.1. Mathematical Literacy

A specification of the general objectives and values with respect to mathematics education can be found in the PISA 2003 assessment framework (OECD 2003). In this document the aim of the OECD/PISA study is determined as the development of "indicators that show how effectively countries have prepared their 15-years-olds to become active, reflective and intelligent citizens from the perspective of their uses of mathematics." (p. 55). The developed assessments focus on the extent to which students can use the mathematics they have learned. Thus the assessment is twofold: it measures the performance of the students but also – and even in the first place - the effectiveness of the educational system. The underlying notion of mathematical competence is that of "mathematical literacy" which is defined as "an individual's capacity to identify and understand the role that mathematics plays in the world, to

[1] Cf. Vollmer 2009 p. 4; CEFR, p. 45.
[2] Cf. the publications of the DeSeCo-project ("DeSeCo" stands for Defining and Selecting Competencies) e.g. DeSeCo 2005.

make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.” (p. 24). Though this definition is focussed on the student's future role as citizen, it is meant in a broader sense which is made clear by terminological explications (p. 25). Taking these explications into account the principal goals assigned to the teaching of mathematics are the following ones:

Mathematics education should enable the students:

to put mathematical knowledge to "functional use in a multitude of different situations in varied, reflective and insight-based ways" (p. 25)
to identify and understand the role that mathematics plays in the "natural, social and cultural setting in which the individual lives" (p. 25)
to make “well-founded judgements” by using mathematics (p. 24)
to use mathematics in ways that meet the needs of that individual's "private life, occupational life, and social life with peers and relatives, as well as life as citizen of a community". (p. 25)
to engage with mathematics through "communicating, relating to, assessing and even appreciating and enjoying mathematics. (p. 25)
…

1.2. Key competences

Mathematics education should also contribute to the development of key competencies which are not specifically mathematical in character. The DeSeCo project names the following - classified in three broad categories and substantiated by the needs of the individual (DeSeCo 2005 p. 10ff):

Using Tools Interactively

The ability to use language, symbols and texts interactively	The need to keep up to date with technologies
The ability to use knowledge and information interactively	The need to adapt tools to own purposes
The ability to use technology interactively	The need to conduct active dialogue with the world

Interacting in Heterogeneous Groups

The ability to relate well to others	The need to deal with diversity in pluralistic societies
The ability to co-operate, work in teams	The importance of empathy
The ability to manage and resolve conflicts	The importance of social capital

Acting Autonomously

The ability to act within the big picture	The need to realise one’s identity and set goals, in a complex world
The ability to form and conduct life plans and personal projects	The need to exercise rights and take responsibility
The ability to defend and assert rights, interests, limits and needs	The need to understand one’s environment and its functioning

1.3. Mathematical experiences

Mathematics as a subject should not only be directed to the development of future competences but should also provide mathematical experiences that can probably only be made in the artificial environment of a mathematics classroom[3]. Among these are:

to perceive and understand the appearances of the world (nature, society, culture) we are concerned with or should be concerned with in a specifically mathematical way.
to understand mathematical objects and states of affairs represented in language, symbols, pictures and formulas as intellectual creations, as a deductively ordered world of its own kind.
to acquire by analysis of tasks problem-solving (heuristic) skills which go beyond mathematics.
to identify mathematics as a part of cultural heritage, as one of the greatest cultural and intellectual achievements of humankind
to experience success by doing mathematics
to experience that mathematics can be interesting and delightful
…

1.4. Linguistic and communicative impact

Even a short reflection on the linguistic and communicative impact of the values, the rights and duties, and the contribution of mathematics as a subject to the general objectives of education (mathematical literacy / competence. key competence, mathematical experiences) should make it clear that their realisation is dependent on linguistic and communicative categories presupposed in these values, rights and duties and general objectives.

That communication, language and language awareness do play a decisive role is most evident with the values of participation in society and of future learning, but can also be recognised as a prerequisite for finding a place on the job market and for the development of personal identity. On the highly abstract level, on which mathematical literacy / competence is formulated, expressions like "knowledge", "identify and understand", "well-founded judgements" and "communicating" already indicate that mathematical competence presupposes and comprises abilities, skills, capacities and competencies in the field of mathematics as well as in the field of subject based language and communication. The key competencies formulated above are also explicitly directed to interaction, language, texts, knowledge, information, communication, working in groups etc. The notion of experience, finally, outlined in the last section encompasses experiences in a narrow sense and their cognitive and verbal reflection.

2. Social and private situations of communication where mathematics plays a role (inventory and description of the social situations of communication involving mathematics in the learners' social environment)

2.1. Situations and contexts where mathematics plays a role

Knowledge of mathematics and its application is needed or at least helpful in many and quite different situations. This is why the PISA 2003 framework stresses the importance of students being able to apply mathematics (and not just have some mathematical knowledge) and that they are able to do so in a variety of situations. Thus most of the test items are located in typical problem-solving situations taken from different domains: "An important aspect of mathematical literacy is engagement with mathematics; using and doing mathematics in a

[3] The first three are due to the German mathematics educator Heinrich Winter (Winter 1995) and reformulated in the German National Educational Standards for Mathematics of the German Kultusministerkonferenz - i.e. German Standing Conference of the Ministers of Education and Cultural Affairs of the Länder (KMK 2004 p. 6). The forth one is mentioned in (NCTM 2000 p. 4).

variety of situations. It has been recognised that in dealing with issues that lend themselves to a mathematical treatment, the choice of mathematical methods and representations is often dependent on the situations in which the problems are presented." (OECD 2003, p. 32).

The PISA 2003 framework classifies the situations and contexts[4] of the test items according to the following two dimensions which focus on the distance of the problem to the student on the one hand and to mathematics on the other hand.

2.1.1. Situation and context considered in terms of the distance between the problem and the student

The problem that is to be solved by applying mathematics may be situated very close to or further away from the student's personal life. Thus, oriented at the domain categories of the CEFR[5]: personal, public, occupational and educational (CEFR p. 45ff.), the following four types are distinguished (OECD 2003, p. 32):

Personal: the student's personal life;
Educational/occupational: school life, work life and leisure
Public: local community and society as encountered in daily life;
Scientific: scientific, also hypothetical scenarios and potential situations[6]

Since these different types of situations/domains are also determined by different linguistic components (register, technical terms, typical pattern, style, types of discourse etc.) the PISA test items in mathematics generally make higher demands to the linguistic competencies of the students than conventional "word problems" (i.e. mathematical problems where significant background information is presented as text). One and the same activity e.g. the activity of selling and buying, can be situated in a personal situation type (selling a bicycle), in an occupational situation type (selling a car as a professional car seller), in a public situation (selling of community property) or in a scientific situation (solving a microeconomic problem). Each of these situation types is determined by genuine linguistic and communicative requirements and conventions and therefore makes different demands with respect to the linguistic-communicative components of mathematical competence.

2.1.2. Situation and context considered in terms of the distance between the problem and the mathematics involved

The problem and the means to solve it may be situated entirely in the "mathematical world" or at some distance to it, either closer or more distant. The distinction explicitly made in PISA 2003 is rather raw ("intra-/extra-mathematical")

intra-mathematical: if it refers only to mathematical objects, symbols or structures, and makes no reference to matters outside the mathematical world
extra-mathematical: if the problem contexts must be translated into a mathematical form

One could arrive at more subtle distinctions in the first category by asking from which kind of mathematics (financial mathematics, mathematics for psychologists, pure mathematics, etc.) the problem comes from, and what kind of mathematical means would be necessary to solve the problem. Since mathematics over the centuries has been used as an ancillary science by

[4] The context (of an item) is explained as "its specific setting within a situation. It includes all the detailed elements used to formulate the problem." (OECD 2003, p. 32)

[5] Council of Europe, Strasbourg (2001): *Common European Framework of Reference for Languages: Learning, teaching, assessment* (CEFR). (Cambridge University Press).

[6] "The use of mathematics to explain hypothetical scenarios and explore potential systems or situations, even if these are unlikely to be carried out in reality, is one of its most powerful features. Such a problem would be classified as belonging to the 'Scientific' situation-type." (OECD 2003, p. 33)

other sciences, differing conceptions have been generated with their own registers and linguistic and communicative conventions. With respect to the second category one could ask from which extra-mathematical sphere the problem arises and whether the problem can be solved entirely by mathematical means or whether other disciplines or considerations (philosophy, ethics, politics, world knowledge, personal experience etc.) are needed in addition.

2.2. A focus on the communicative aspect of situations and contexts where mathematics plays a role

The classification of situations in terms of the distance between the problem and the student / the mathematics involved in the PISA 2003 framework is motivated by the differing social and private situations of communication in the real world. Mathematical items of the PISA tests, however, only contain descriptions of such situations. Solving mathematical word problems in a paper and pencil test thus differs substantially from solving these problems in a real communication situation.

But, nevertheless, since these items are constructed as indicators for the ability of the students to use mathematics in real situations, we can refer to these items as describing typical examples for social and private situations of communication where mathematics plays a role. Thus the distinction between personal, educational/ occupational, public and scientific can be made in two directions: (i) directed to the situation where the problem is located and/or (ii) directed to the situation / the discourse community where the problem is posed or (re-)formulated and solved.

The role that mathematics can play in situations of communication is of course not confined to problem solving. In fact, in most cases mathematics can only give a contribution to such a solution, or it can give support to a better understanding, e.g. by sketching a geometric arrangement, by visualising a connection, by drawing a diagram with some spreadsheet software etc. Properties of all kinds are measured and expressed by numbers whenever it seems possible in scientific as well as in everyday contexts. Connections of different sorts can be interpreted as mathematical functions and thus give rise to a functional understanding of the world.

Since mathematics is used in science, technology and day-to-day life as an ancillary science and a means of understanding, there is also a need to learn more in mathematics and mathematics related subjects like ICT for occupational, educational or personal reasons. Hence there are private or social and formal or informal situations, far away from obligatory school, where the learning of mathematics is the focal point. Other persons with advanced mathematical competencies, though not mathematics teachers, are expected to give support in situations where the learning of mathematics is at stake – e.g. parents are expected to give support to their children, peers to peers, colleagues to colleagues, partners to partners. Thus there are teaching or support situations as a complement to the learning situations.

Among the various situations of communication in which mathematics is or could be used are the following:

Personal situations: situations within Informal social settings: family, peers, friends

discussing, posing and solving problems by means of mathematics
… that are close to the informal social setting: e.g. talking about mobile contracts and taking a decision between two offers.
… that are further away from the informal social setting: e.g. discussing a newspaper article about competing financial strategies proposed by political parties
…

trying to understand and to explain a development, a functional connection, a geometric arrangement etc. … which are close to the informal social setting: e.g. sketching the floor plan for arranging a flat. … which are further away from the informal social setting: using mathematical operations in order to reconstruct the possible meaning of a newspaper article where absolute and relative numerical data are mixed. …
learning, teaching, explaining, exercising mathematics … where the subject is close to the informal social setting: e.g. learning how to change a recipe for a cake. … where the subject is further away from the informal social setting: e.g. parents helping their children to understand a mathematical problem, notion, operation …

Occupational situations: situations within formal social settings: colleagues, superiors, customers,

solving problems in a team by means of mathematics …that are close to / further away from the occupational sphere: e.g. building or constructing a house, an engine, a machine; making a budget plan …
…
trying to understand and to explain a development, a functional connection, a geometric arrangement etc. … which are close to / further away from the occupational sphere: e.g. interpreting a development or a state of affairs based on a statistical data analysis …
…
learning, teaching, explaining, exercising mathematics … where the subject is close to/ further away from the occupational sphere: e.g. attending an upgrade training course, where mathematical methods are prominent. …

Public situations: situations within formal social settings: offices, business partners, customers

discussing, posing and solving problems by means of mathematics … that are close to / further away from the formal social setting: e.g. creating a business plan …
trying to understand and to explain a development, a functional connection, a geometric arrangement etc. … which are close to / further away from the formal social setting: e.g. comparing different offers …
learning, teaching, explaining, exercising mathematics … where the subject is close to / further away from the formal social setting: e.g. explaining a calculation / a special algorithm in financial mathematics to a customer …

Scientific situations: situations within formal social settings: mathematicians, other scientists, mathematics educators,

discussing, posing and solving problems by using mathematics … in pure or applied mathematics, … in other sciences e.g. physics, … in technology e.g. ICT, engineering

…
trying to understand and to explain a development, a functional connection, a geometric arrangement etc. … e.g. explaining the differences in educational systems by means of probabilistic test theory …
learning, teaching, explaining, exercising mathematics … e.g. attending a scientific congress in the field of mathematics …

Corresponding to these distinctions of social situations where mathematics plays a role, one can easily construct an inventory of private situations, which are in spite of their privacy determined by the registers of the different domains (personal, educational / occupational, public, scientific, …)

The situations mentioned involve different forms of communication: oral, written and audiovisual reception, oral and written interaction, oral and written production. The list may be supplemented and used as a guide to the identification of language skills forming part of mathematics syllabi. The social activities involving mathematical knowledge and competencies can be described in terms of discourse types and linguistic capacities.

Like in other school subjects, goals and types of discourse have been developed in the history of mathematics education which are situated more specifically in education and classroom contexts. These are dealt with in part 3. These educational goals and practices form a dynamic body in the history of institutional learning. Also, their link to situations outside the classroom varies and changes over time.

2.3. From social situations to types of discourse

For situations of mathematics communication it is possible to develop descriptors based on an analysis of the characteristics of the types of discourse employed in those situations. One example is analysed in more detail. First the cognitive skills underlying the discourse are spelt out, followed by the linguistic and semiotic skills which cover the language-driven activities. The example concerns the social situation, in which mathematics is used as a contribution to solve a real-world problem. The description of activities in "mathematical processes" is taken from the preprint version of the PISA 2012 framework (OECD 2010 p. 14ff.)

The mathematics-related cognitive skills include the ability to …

… formulate situations mathematically[7]:

Identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables
Recognising mathematical structure (including regularities, relationships, and patterns) in problems or situations
Simplifying a situation or problem in order to make it amenable to mathematical analysis
Identifying constraints and assumptions behind any mathematical modelling and simplifications gleaned from the context
Representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models
Representing a problem in a different way, including organising it according to mathematical concepts and making appropriate assumptions
Understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically

[7] cf. OECD 2010 p. 14f.

Translating a problem into mathematical language or a representation, i.e., to a standard mathematical model
Recognising aspects of a problem that correspond with known problems or mathematical concepts, facts, or procedures
Using technology (such as a spreadsheet or the list facility on a graphing calculator) to portray a mathematical relationship inherent in a contextualised problem
…

… employ mathematical concepts, facts, procedures, and reasoning[8]

Devising and implementing strategies for finding mathematical solutions
Using mathematical tools, including technology, to help find or approximate solutions
Applying mathematical facts, rules, algorithms, and structures when finding solutions
Manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations
Making mathematical diagrams, graphs, and constructions and extracting mathematical information from them
Using and switching between different representations in the process of finding solutions
Making generalisations based on the results of applying mathematical procedures to find solutions
Reflecting on mathematical arguments and explaining and justifying mathematical results
…

… interpret, apply and evaluate mathematical outcomes[9]

Interpreting a mathematical result back into the real-world context
Evaluating the reasonableness of a mathematical solution in the context of a real-world problem
Understanding how the real world impacts the outcomes and calculations of a mathematical procedure or model in order to make contextual judgments about how the results should be adjusted or applied
Explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem
Understanding the extent and limits of mathematical concepts and mathematical solutions
Critiquing and identifying the limits of the model used to solve a problem
…

The activities described above concern private as well as social situations of problem solving. In the first case "thinking aloud" would exhibit the close connection between cognitive and linguistic activities (cf. Section 5). In the second case communicative activities accrue for which B1-B2 competencies described under the "heading goal–oriented cooperation" in the CEFR (CEFR, p. 79) are relevant, if they are reformulated as plurilingual competencies.

Linguistic and communicative skills include the following abilities ...

Can understand detailed instructions reliably. B2
Can help along the progress of the work by inviting others to join in, say what they think, etc. B2
Can outline an issue or a problem clearly, speculating about causes or consequences, and weighing advantages and disadvantages of different approaches. B2
Can follow what is said, though he/she may occasionally have to ask for repetition or clarification if the other people's talk is rapid or extended. B1

[8] cf. OECD 2010 p.16
[9] cf. OECD 2010 p.17

Can explain why something is a problem, discuss what to do next, compare and contrast alternatives. B1
Can give brief comments on the views of others. B1
Can generally follow what is said and, when necessary, can repeat back part of what someone has said to confirm mutual understanding. B1
Can make his/her opinions and reactions understood as regards possible solutions or the question of what to do next, giving brief reasons and explanations.
Can invite others to give their views on how to proceed.
…

3. Mathematical competencies (inventory and description of some basic /the expected mathematical knowledge structures)

The values, the rights and duties and the objectives related to mathematics education have been exposed in the first section, and the private and social situations where mathematics plays a role have been expounded on in the second section. The mathematical competencies, abilities, skills, etc. that are valued as important for the personal welfare of the individual and for the public welfare of society are systematically organised into competency models, which describe the relevant dimensions of mathematical competence and their relations to each other, among others: areas (contents), aspects (processes), levels, the developmental trajectories of competencies etc.

3.1. An example of a mathematical competency model

The mathematical competency models used as a basis for National educational standards in European countries differ in terminology and also in their conception due to historical, political and organisational reasons. The categorisation used in the following text is taken from the mathematical competency model of the Swiss National educational standards.

Content dimension:

Number & Variable
Shape & Space
Functions & Relations
Size & Measurement
Data Analysis & Probability
…

Process dimension:

Knowing, Recognising & Describing
Operating & Calculating
Using Instruments & Tools
Presenting & communicating
Mathematising & Modelling
Arguing & Justifying
Interpreting & Reflecting on Results
Experimenting & Exploring
…

Combining the two dimensions one arrives at a grid with 40 cells each containing a description of one or more competencies. On a more abstract level it is sufficient to cluster the competences belonging to the same process dimension:

Among the mathematical competencies that are valued as important for each child at the end of obligatory school are the following:

Knowing, Recognising & Describing

understanding, using and explaining technical terms
relating technical terms to mathematical objects, properties and relations and vice versa
identifying forms and pattern
naming and describing mathematical rules and laws in their own language
capturing mathematical states of affairs and describing them
…

Operating & Calculating

Carrying out calculations, transformations and constructions in written "standard form", with notes or orally, with or without (technical) instruments
…

Using Instruments & Tools

Using electronic instruments (calculator, Computer), works of reference, construction instruments (compass, set square)
…

Presenting & communicating

Understanding calculations, transformations, constructions, argumentations of other students
Presenting own calculations, transformations, constructions, argumentations in a way that is comprehensible and traceable by others and appropriate with respect to the mathematical object
,,,

Mathematising & Modelling

Describing, interpreting and modulating (problem) situations (of daily life) in order to solve them by mathematical means
,,,

Arguing & Justifying

Forming assertions and giving reasons for them
Making thoughts and ways of calculating transparent and justifying them
Giving illustrative explanations for mathematical phenomena and laws
understanding and reproducing proofs and counterexamples
,,,

Interpreting & Reflecting on Results

Checking results for truth and for adequacy with the original problem
Reflecting whether a result or an approach can be used for future problem solving
,,,

Experimenting & Exploring

Exploring mathematical situations and searching for mathematical laws
Expressing conjectures and trying to support or falsify them by thought experiments
,,,

3.2. Linguistic and communicative competences as components of mathematical competence at different stages.

The educational values targeted by mathematics teaching practices can only be realised if linguistic and communicative competence is also targeted at the same time, since

Linguistic and communicative competencies are preconditions for learning: Students can only successfully participate in learning mathematics if they have the linguistic prerequisites to understand questions, problems, argumentations, etc. and are able to give answers, to interact with others, etc.
Linguistic and communicative competencies are constitutive parts of educational standards in mathematics: Students can only reach the educational standards in mathematics at the end of compulsory school if they are able to name, describe, define, explain, argue etc.
Linguistic and communicative competencies are preconditions for making experiences, for mathematical literacy, for key competences: All these presuppose a certain degree of reflection, which can only be realised on a higher level of language competence

4. In-school communication situations relating to mathematics teaching and learning

We now have to switch the focus from communication in society and from the objectives defined in terms of mathematical knowledge and procedural competence to the *types of teaching and learning in school.* The latter have to be informed by the former: the forms of communication that are used in mathematics education must be linked to those present outside school. Yet, school-based education also follows its own rules and conventions.

In general, we can distinguish between several different phases or types of learning activities in the classroom, and this is also true for mathematical education. Each of them involves different cognitive-linguistic demands and challenges.

4.1. Checklist of classroom activities in mathematics education (for subject teaching / learning in general)

The forms of teaching and learning mathematics vary according to educational traditions and the methodological choices made in the syllabi or by individual teachers, all of which structure the processes of teaching and learning. Most of the forms used in mathematics classrooms are also used in other subjects such as history, science or literature[10], but there are also others that are more peculiar for mathematics - especially a combination of oral and written interaction “OWI” using mathematical symbols, sketches, grids etc. – cf. CEFR (Council of Europe 2001, p. 82 4.4.3.3 and p. 90 4.4.5.3). Among the different forms of teaching and learning mathematics are:

[10] Cf. Beacco, Coste, van de Ven, Vollmer (2010, p. 12-14). The Coding of communication activities is based on the CEFR: R = reception; P = production; I = interaction; M = mediation; O = oral; W = written.

Presentation by the teacher (teacher-learner interaction as monologic instruction) using visual aids (maps, diagrams, data tables, graphs, computer animations, applets dynamic geometry software etc.) (AuR, WR and WP note-taking)	teacher-learner interaction as monologic instruction / frontal education
Teacher-learner interaction as dialogic instruction (OI)	teacher-learner interaction as dialogic instruction / pedagogical dialogue / IRE model[11]
Learners presenting the results of their homework using visual aids (OWP), comparing the results (AuR, WR), asking and answering questions using visual aids (OWI)	learner-learner interaction as monologic instruction / presentations by students
Learners explaining a mathematical conception, an assertion, a rule, a procedure, a proof etc. to others (OWI)	learner-learner interaction as dialogic instruction / instruction by peers
Learners read the textbook (WR) and solve problems (WP) or work individually within a learning environment (WR) and (WP)	Individual work (problem solving)
Learners working on mathematical exercises individually (WP)	Individual work (practising)
Learners exploring individually a mathematical state of affairs or testing conjectures systematically (WR, WP)	Individual work (exploring)
Gathering information (WR and WP: note-taking);	Individual work (gathering information)
Activities run as projects (linking different competences) of individual research; e.g. inventing new mathematical problems (WP) departing from solved problems (WR)	Individual work (working on an individual project / creative work)
Learners writing a study diary (WP)	Individual work (metacognition)
Learners interact with fellow pupils in group work (WR, WP, OI, OWI) solving a problem or working together with a learning environment	Group work (problem solving)
Learners exploring individually a mathematical state of affairs or testing conjectures systematically (OWI , WP)	Group work (exploring)
Searching information: planning the search (OI), gathering information (WR and WP; note-taking); sharing the results (OWI)	Group work (planning the search for information), individual work (gathering information)
Presentations of the results of group work by learners (OP) based on notes, using PowerPoint (O&WP), Blackboard (O&WP), etc.; answering questions (OWI)	Group work (preparing a presentation), individual work (presenting)
Activities run as projects (linking different competences) as group research (OWI, O&WP);	Group work (working on a common project)
Teamwork: Developing new mathematical problems (OI, WP);	Group work (creative work)
Controlling and reflecting results (WR, OI)	Group work (reflective work)
[...]	

All of these forms of activities which can be found in mathematics classrooms – though perhaps some of them are more common than others - have been established as instruments to support the development of mathematical competence. The indicated type of communicative language activities is to be understood as part of the mathematical competence that has to be developed. In this sense classroom activities can be understood as an anticipation of real life activities in the future that can be tried out without severe consequences, if they go wrong. On the other hand a certain degree of linguistic and communicative competence is a prerequisite for the participation in these activities and thus for learning - just in the same sense as a certain degree of mathematical content knowledge and competence is a prerequisite for successful learning in mathematics. Students can only

[11] The IRE model (Initiation by the teacher – Response by the learners – Evaluation by the teacher) was presented by Sinclair & Coulthard (1975).

successfully participate in such learning situations if they get the necessary scaffolding comprising mathematical content, as well as linguistic and communicative knowledge and competencies.

4.2. From classroom situations to discursive forms

These types of mathematics teaching and learning activities can be described in terms of linguistic capacities and types of discourse and it is possible to develop descriptors from the characteristics of the discursive style used in those situations.

Example: Giving a (prepared) presentation to the class using auxiliary means (data projector, blackboard, flipchart, overhead projector, etc.) to visualise a mathematical derivation, construction, procedure, calculation etc.

Presentations using auxiliary means for visualising a line of thought are typical for mathematics. They constitute a type of discourse that relates to oral production (see CEFR, 4.4.1.1.: addressing audiences) based on notes, slides or a whole manuscript in written form, but also to written interaction (see CEFR, 4.4.3.2 and 4.4.3.2). This involves among others the ability to:

State a plan, a scheme of presentation;
"Give clear, systematically developed descriptions and presentations, with appropriate highlighting of significant points and relevant supporting detail." (Overall OP: descriptor B2 in the CEFR, p. 58);
"Give clear, systematically developed descriptions and presentations on a wide range of subjects related to his/her field of interest, expanding and supporting ideas with subsidiary points and relevant examples." (Overall OP: descriptor B2 in the CEFR, p. 58);
Emphasise the stages of the presentation as it unfolds;
Present and organise the linguistic commentary of tabulated data, a diagram, etc.;
"Convey information and ideas on abstract as well as concrete topics, check information and ask about or explain problems with reasonable precision." (Overall WI: descriptor B1 in the CEFR, p. 83);
Make the presentation attractive: manage voice and intonation;
React with restraint to objections or criticism from class or teacher;
"Can depart spontaneously from a prepared text and follow up interesting points raised by members of the audience (…)." (Addressing audiences OP: descriptor B2 in the CEFR, p. 58)
Answer questions concerning the findings and/or the procedures applied afterwards;
Assess one's own performance (without or with the help of others);
[...]

It will be noticed that in the example presented above descriptors of the CEFR, devised for foreign languages, can be used or easily adapted for the description of discourse types in mathematics classrooms, but of course not all of the descriptors are relevant. Likewise, the level C1 and C2 descriptors can sometimes furnish material for descriptions but probably cannot be adopted on the whole.

Plurilingual education presupposes linkage of the classroom modes of communication to the social ones involving science, so as to make transfers of proficiencies between them. At least some of the classroom modes of communication should enable learners to handle social situations of communication with mathematical content:

- *either directly through the classroom use of these social forms*
- *or indirectly, with the same proficiencies as those inherent in the social forms being developed through the classroom forms.*

5. Specific linguistic and semiotic competences needed for mathematics education

So far we have identified and exemplified

- social situations and contexts where mathematics plays a role (2.1. checklist)
- the communicational aspects of such situations and contexts (2.2.)
- an example (problem solving according to PISA 2010) of the corresponding types of discourse (2.3 checklist)
- an example of a mathematical competency model (3.1. checklist)
- linguistic and communicative competences as components of mathematical competences at different stages (3.2. checklist)
- Classroom activities in mathematics education (4.1. checklist)
- the corresponding types of discourse in mathematics lessons in school (example in 4.2.).

Based on these different steps (and their underlying principles) it is now possible to single out and generalise specific linguistic competences suited for mathematics teaching and learning, aimed at imparting knowledge and expertise as well as instilling social communication skills. As already demonstrated, for learners these cannot be restricted to command of specialised terminology or the ability to piece together elements of mathematical knowledge, even where these may be clear and logically derived from data. The necessary linguistic competences involved in mathematics education also involve complex thinking and discourse skills and ways of relating the two via lexical, grammatical and textual choices.

To describe these linguistic and communicative competences in more general terms, we shall adopt a subject-based model of *capability* and *communication,* arranged in four sets of components, the first three of which form what is strictly speaking linguistic communication competence:

- strategic component/competence (see 5.1.)
- discursive component/competence, mastering types of discourse) (5.2.)
- formal component/competence (5.3)
- interdisciplinary/cross-curricular competences, not peculiar to mathematics teaching: these will have to be dealt with in another module.

5.1. Strategic competence[12]

General communicative ability includes a psycho-cognitive component termed *strategic* that controls observable linguistic behaviour in order to generate, produce and understand texts. “Strategies are a means the language user exploits to mobilise and balance his or her resources, to activate skills and procedures, in order to fulfil the demands of communication in context and successfully complete the task in question in the most comprehensive or most economical way feasible depending on his or her precise purpose.” (CEFR, p. 57).

In the CEFR the strategies are situated at the same level as communicative activities (as oral/written interaction [OI/WI], oral/written production [OP/WP] and aural/written reception [AuR/WR]). This level of specification allows teachable actions to be defined in terms of planning, execution, evaluation and repair[13], which seem independent of the languages and discourses used. We shall proceed from these specifications to describe the communication proficiencies needed to teach/learn mathematics.

[12] This paragraph is taken with minor adaptations from Beacco (2011) and Vollmer (2011).
[13] CEFR, 4.4.1.3. for OP/WP, 4.4.2.4. for OR/WR and 4.4.3.3. for OI/WI.

Oral and written production[14]

	General activities	Activities in the school setting of mathematics teaching and learning
Planning	Locating resources Preparation and/or rehearsal Consideration of the recipient and audience Adaptation of message	Identifying the relevant information sources Producing successive tentative versions of the text to be produced. Verifying its length (if WP). Taking account of the audience's receptive capabilities, level of knowledge and status, etc., Transposing, paraphrasing, summarising, mentioning, quoting and commenting on source texts
Execution	Building on prior knowledge Trial (experimentation)	Reliance on existing texts of the same kind as the one contemplated Making successive provisional versions of the text to be produced.
Evaluation	Checking of results	Testing through listeners' reactions (if OP) the intelligibility to an outsider not directly addressed (if WP)
Repair	self-correction	Improving self-correction through an external evaluation

Aural and written reception[15]

	General activities	Activities in the school setting of mathematics teaching and learning
Planning	Framing (selecting mental set, activating schemata, setting up expectations)	Identifying type of discourse and its potential contents
Execution	Identifying cues and making inferences	Working out the meaning of technical terms or mathematical deductions from language knowledge and knowledge in mathematics
Evaluation	Hypothesis testing: matching cues to schemata	Matching up the interpretative hypotheses and developing critical sense
Repair	Revising hypotheses if required	Reconsidering one's position about a theory, explanation, validity of data and their interpretation

It is obvious that the specifications of the CEFR relate more to reading as comprehension than as interpretation or critical response. For languages of instruction, the comprehension strategies need to be re-interpreted as a function of the knowledge in the discipline (in this case, critical comprehension).

[14] According to the CEFR, p. 53.
[15] According to the CEFR, p. 65.

Spoken and written interaction

	General activities	Activities in the school setting of mathematics teaching and learning
Planning	Framing the issue (establishing a line of approach) Judging what can be presupposed Planning moves	No relevant descriptors in the CEFR, since the interactions between teacher and learner or among learners occur in the language of schooling. But it is necessary to understand what is expected of the classroom interactions whose aim is to provide insight into the knowledge presented and which are not ordinary social interactions. It is thus important to know their implications for imparting knowledge.
Execution	Taking the floor Co-operating (interpersonal) Dealing with the unexpected Asking for help	These specifications are altogether relevant in the context of debates, discussions and arguments staged in class about mathematical questions
Evaluation	Monitoring (schema, praxeogram) Monitoring (effect, success)	No particular specificity to the mathematics-related verbal styles in or out of class
Repair	Asking for clarification Giving clarification Communication repair	Relevant as regards terminology, foreign borrowings, knowledge and patterns of scientific reasoning and explanation...

These descriptors of strategies, as may be seen, need specifying if possible, as far as types of communication with “mathematical” content are concerned. This reference grid should therefore be considered provisional. From a pedagogical standpoint, the descriptors of planning, which relate to the learners’ preparation of the statements (oral or written) should no doubt be more developed than those concerning monitoring or correction (except in the case of OP or WP).

These strategic abilities are valid for all subjects taught, so a comparison with the terms in which they are specified for history, science or art (e.g.) is called for.

5.2. Discursive competence

The concept *type of discourse* (or *discursive form*) has been used to denote the forms taken by communication as practiced in a given social situation and communication community. The types of discourse are specific discursive forms identified as such by a standard name and certain characteristics (physical location, type of participants, medium, etc.) of the situations where they occur: lecture, news item, observation, dispute, myth or prayer, etc.

The texts that pertain to a given type tend to follow the conventions typifying these discourses; the conventions concern not only contents but also the structure and/or verbal forms of realisation/productions. A text is more or less consistent with the discursive form whose specific outcome it is. The types of discourse themselves are more or less strained and formalised (lecture *versus* casual conversation).

The concept of discourse type is less abstract than that of textual type (narrative, descriptive, imperative, expository, persuasive, etc.). Typologies of this kind have never really been adequate for describing classes of texts since it is readily acknowledged that *most actual texts correspond simultaneously to several types*. This typology may nevertheless be used to denote the style (or discursive regime) adopted by certain segments of texts: for example, in the “film/book/record/review” discourse type in written media, there is often a segment at the beginning which has a descriptive or narrative tone (film); the texts then continue with a segment with an evaluative purpose, before summarising and highlighting the main points.

One aim of plurilingual and intercultural education, hence of languages in teaching/learning, is to broaden learners' discourse repertoires (in some/all of the languages of their language repertoires) in relation to their initial experience/proficiency in types of discourse and to give them the opportunity for new experiences (through texts and documents including non-verbal forms of representation) of the diversity of disciplines, academic cultures and of otherness.

As in every other subject, mathematics syllabi may be specified according to discourse type:

- types seen as already entering into the learners' repertoires (textbooks; learning environments; internet sites offered for students by students, teachers or institutions; social networks, wikis, internet groups; scientific documentaries; illustrations and animations of (abstract) relationships and functions, info brochures; discursive forms in other subjects, in which mathematical procedures or ways of presentation are used; mathematical games, puzzles, riddles, etc.)
- types present in the learners' social environment (periodicals: general-interest press, science-based journals; websites, applets, dynamic geometry software, computer algebra systems, instruction manuals where mathematics is used; expert debates, moderated public and/or political discussions, etc.)
- types to which a certain form of exposure is sought by mathematics teaching.

For the purpose of choosing the types of discourse with which learners are to be familiarised, attention needs to be paid first to the academic status of statements of "facts" and of popularised reports concerning science and mathematics. These are very diverse in nature because of the role assigned to them in diverse texts in the public domain, which have some connection with mathematics and the natural sciences. For example, with respect to written scientific reports, it may be deemed important for learners to be brought into contact with:

- academic/disciplinary discourse types written by specialists for specialists (articles, communications, monographs, theses and the like) in mathematics or other sciences and disciplines where mathematics plays a role;
- types produced by specialists, presenting new knowledge meant for and made accessible to the ("educated") general public;
- types used in popularisation in book form or as TV features by professional scientists, knowledgeable amateurs and authors specialised in scientific dissemination;
- journalistic discourse types of the press specialising in issues of mathematics and natural sciences;
- journalistic discourse types of the ordinary daily press relating to scientific and mathematical questions, procedures and debates (reviews of published books, accounts of "discoveries", interviews with mathematicians, scientists, with interested laymen, etc.);
- educational discourse in the form of textbooks in mathematics or other subjects where mathematics is used, other summaries for school learners, multi-media presentations on film or video;
- educational discourse in the form of popularised mathematics textbooks, auxiliary learning material and games;
- the encyclopaedic discourses of formularies, dictionaries, encyclopaedias, Wikipedia / the internet in general, etc.;
- the direct testimonies recorded for example in autobiographies, recollections and personal diaries, statements of representatives of interest groups, etc.;
- fictional or "literary" works of a scientific or mathematical nature: novels, films, TV series, etc.;

The choice of the discourse types which it is considered learners should experience and partly even produce (either by way of simulation or by way of (local) participation), depends on the general choices already described above (values, social situations of communication, mathematical and scientific knowledge, status of knowing, controversies involved, etc.) but may be fine-tuned in the light of descriptors relating to:

- the nature of the instructional / learning activities, which are to draw upon these texts (WR, OI...)
- the expected degree of competence or proficiency for each (see sections 3 and 4)
- the proximity or familiarity of the types compared to those already experienced by the learners
- the interest (or motivation), which these discourse types may arouse
- the necessity of dealing with certain discourse types due to their importance and impact outside school.

Characteristics peculiar to the discourse types may also be used as a basis for decision-making on the following levels:

- length of the texts pertaining to them
- predictability (as to layout, form of paragraphs and phraseology)
- complexity (number of items linked, nominalisations, hypertactic constructions)
- use of explicit headings and subheadings, summaries, etc.
- use of graphics, illustrations, maps, diagrams, etc.

These inventories lend themselves as a basis for decision-making about the discourse types suitable for mathematics education in school and as a checklist for evaluating the traditional materials and discourse types used so far in different parts of Europe. The inventories are helpful and appropriate to guide choices in planning curricula and compiling teaching programmes, which may differ, yet which are based on similar categorisations of discursive forms.

5.3. Formal competence

In addition to lexical/terminological and discursive competences a more formal competence of handling the macro and micro structures of the discourse types plays a decisive role: this involves the capability of linguistic expression of cognitive processes underlying the analysis (comprehension) and the construction (production) of concrete discursive forms (or texts).

5.3.1. Pragmatic and cognitive categories

The conventions of form recurring in types of discourse (i.e. the linguistic and structural deliveries of the texts) may thus be described by means of categories unconnected with the syntax of the sentence.

These may be categories like speech acts/language functions or, on a higher, more abstract level, *discourse functions*. These analytical categories applied to texts (and also or alternatively to the *cognitive processes*) are to be understood as the discursive representation of both the cognitive processes and their linguistic realisation (in the sense of enactment) brought into play for the development/exposition of knowledge.

These discourse functions mark cognitive operations *and* their verbal performance at the same time; they are at the interface between cognition and verbalisation, they include operators (or terms) such as[16]:

analyse	*define*	*correlate/contrast/match*
argue	*distinguish*	*name*
assess	*enumerate*	*outline/sketch*
calculate	*explain*	*prove*
classify	*illustrate/exemplify*	*recount*
compare	*infer*	*report (on) a discourse*
describe/represent	*interpret*	*summarise*
deduce	*judge/evaluate/assess*	*specify* [...]

Among these many discourse functions, there are some which are more basic or comprehensive and relatively distinct from one another in terms of cognitive operations and discursive forms involved (they might be called *macro functions*), while others may appear under several macro functions and serve a number of them, not just one (these might be called meso and micro functions – for our purposes we just refer to them as *micro functions*).

Among the macro functions, there are at least the following ones:

1. Exploring/processing/documenting
2. Naming/defining
3. Describing
4. Reporting
5. Explaining
6. Evaluating
7. Arguing
8. Exchanging / negotiating
9. Narrating
10. Creating
11. Reflecting (e.g. about learning paths + results)
12. Acting (symbolically or by way of simulation)

Each macro function is served by a great number of micro functions. Among the many micro functions, we could list the following ones:

Asking questions	Labelling	Presenting	Hypothesizing
Questioning	Collecting	Sequencing	Predicting
Guessing	Selecting	Relating	…
Identifying	Reporting	Structuring	
Classifying	Summarizing	Contrasting	

These micro functions operate on a lower level than the macro discourse functions, but they also describe and specify both cognitive *and* verbal activities at the same time.

As to mathematics education, to describe academic discourse in this subject area all of the macro functions mentioned above would play an important role, whereas a specific subgroup

[16] See the extended list in Vollmer et al. 2008 which was arrived from the analysis of modern science curricula (and other subjects) for grade level 9/10 in Germany. See also the set of Macro-functions derived from this analysis (Vollmer 2009, updated Vollmer 2011).

of cognitive/discursive operations/processes on the micro level would be prominent in this context such as:

- *reporting / recounting* (on a solution of a problem, an exploration)
- *classifying* (mathematical objects, properties, relations, procedures)
- *defining / determining* (a mathematical term, a mathematical state of affairs)
- *representing* (mathematical objects, relations or data)
- *interpreting* (a mathematical state of affairs, the results of a calculation)
- *matching* and/or *contrasting* (problem situation and the solution found)
- *deducing* (conclusions from data)
- *justifying* (steps in a solution, the chosen approach / procedures, decisions)
- *embedding* (an observation / a finding into a larger structure)
- *reflecting or weighing* (learning paths, arguments for and against a decision)
- [...]

For each of these operations it is possible to identify the linguistic resources needed for their enactment, with likely variation between discourse types. It may be assumed that the above "words" (verbs, verbal operators) referring to cognitive operations have equivalents in all languages and that an attempt could be made to compile transposable inventories (for different languages and different subjects).

To compile such inventories of forms required to express the cognitive-discursive operations occurring in given types of discourse, one ought to use again the *Descriptions of language-specific reference levels in the CEFR*[17]

5.3.2. Linguistic categories for the description of discourse types

Discourse types can be described by using speech acts and/or cognitive operations or, as suggested here, by using *discourse functions* which link cognition and verbalisation, since a specific discursive form is a verbal object, yet governed by cognition underlying it. Discourse functions (on the macro as well as on the micro level) are distinct from utterance, text, speech act, type of text, etc.; their verbal conventions may be apprehended

- *as relatively stable types of utterances, in the case of highly restrictive types, set phrases, etc.*
- *as the relatively stable or predictable general scheme or elements of their structure, which may be broken down into stabilised successions of speech acts or cognitive operations (for example, the series: represent, interpret, match...)*
- *as the preferential forms, in a given type, with which to deliver them. This conformity determines the appropriateness of the utterances (and not their accuracy or grammatical correctness), that is their compliance with common "rules" on the acceptable makeup of discourse types.*

These conventions may be described on the basis of various general linguistic categories (= independent of individual languages), such as:

- *forms of actualisation of the speaker (for example, in English: I/me, we, one, impersonal, passive, reflexive, etc.);*
- *forms of actualisation of the person addressed;*
- *presence/distribution and expected forms (in a given type) of assertive, appreciative, ethical and other formulations;*

[17] Available, or being produced, for English, German, Spanish, French, Greek, Portuguese...

- *presence/absence/distribution and forms of meta-discursive indications (statement of text plan, relating to known rules, assumption of prior knowledge, etc.);*
- *standard form of certain paragraphs;*
- *discursive tone (serious, humorous, personal touches, etc.).*
- *[…]*

All descriptive categories used when analysing a discourse may serve as a starting-point for descriptors of formal mastery, especially with respect to reception or production. Nonetheless it has to be taken into account that:

- *texts of the same discourse type comply to varying degrees with the (often unstated) model underlying it;*
- *discourse types themselves may be conventional to varying degrees either as a whole or in some of their constituent parts (for example, the beginnings of scientific articles may be quite conventional/predictable while those of newspaper articles are fairly unpredictable).*

This specification of forms should be underpinned by the expected language skills in other subjects taught and in language as a subject.

5.3.3. Executing discursive competence: two examples

First example: to state a plan (in OP) – with descriptors such as:

The learner is able to produce (W or O) a statement of plan appropriate to the types under consideration (here, presentation to the class), by activating some of the following linguistic resources:

[Highlight the structure of the forthcoming discourse]

[statement of the general schema]

- I am about to speak of/examine/deal with the question/the issue of...
- I shall talk about...
- My topic is…

[each point introduced by means of cohesive devices]

- first of all, first, to begin with, etc.
- next, then, as the second point
- the following point
- ...
- a final aspect is…, etc.

[announcement of the end]

- lastly, to conclude, to finish, in conclusion

Second example: reporting about a mathematical exploration/experiment (W) – with descriptors such as:

The learner is able to describe (W)

- the aims of the exploration/experiment,
- the means chosen,

- the observations made
- the results obtained and
- what could be expected as a generalisation

by activating some of the following linguistic resources:

[Highlight the structure of the forthcoming discourse: Introduction]

- In the following, I will write/report about…
- The report is about…
- My topic is about…

[statement of the general goal of the mathematical exploration/experiment]

- The purpose of the exploration/experiment was… to find out whether/in how far/…how much…
- The exploration/experiment was to examine/to deal with the question/the issues) of...
- In the exploration/experiment I looked at...
- We were asked to explore… /observe… /find out…

[name necessary subtopics like: choosing the appropriate means, carrying out the exploration / performing the experiment, observing and documenting regularities and irregularities; each point introduced by means of paragraphs with cohesive devices]

- first of all, first, to begin with, etc. was set up…
- next, then… sth. was started/initiated…
- the next step was ….
- An observation sheet had been prepared for…
- It could be observed that…

[announcement of the results of the exploration/experiment, possible conjectures, the necessity of proving and/or of the end (of the report)]

- As a result (we can say/we have…)/ It showed that…/
- The exploration/experiment showed that…
- As to the goal of the exploration/experiment, …
- As a possible generalisation it could be expected that …, but … / A possible conjecture could be …
- Could we prove that … / It seems to be certain, that …, but can we prove it?
- Finally,…, etc.
- To conclude, to finish, in conclusion…

Inventories of this type may be common to different languages and to different disciplines in some respects, but they necessarily comprise language-specific realisations owing to their morphological and syntactic structures and the diversity of discursive forms in the classrooms, in a country, in a discipline.

6. Summary and Perspectives: Thresholds and stages of development

Users are invited to determine from the categories set out above which thresholds of knowledge and language skills (concerning mathematics-related discourse types) the learners should possess, according to:

- expected proficiencies (OI, OP...)
- types of discourse to be mastered (for reception or production)

- cognitive operations or discourse functions which they must be able to recognise or deliver
- forms for delivering the above, which they must know how to handle correctly and suitably.

Empirical analyses of correct applications of these forms in productions (examination papers, for example) by learners who have taken courses of this kind or in other types of discourse to which they are exposed (textbook) should make it possible to estimate whether the results are actually achieved, hence to judge realistically whether they are within the learners' grasp.

7. Selected bibliography

Beacco, J.-C. (2010). Items for a description of linguistic competence in the language of schooling necessary for teaching/learning history (end of obligatory education). Retrieved from: http://www.coe.int/t/dg4/linguistic/Source/Source2010_ForumGeneva/1_LIS-History2010_en.pdf (7 November 2012)

Beacco, J.-C.; Coste, D.; van de Ven, P.-H., and Vollmer, H. (2010) Language and school subjects. Linguistic dimensions of knowledge building in school curricula. Retrieved from: http://www.coe.int/t/dg4/linguistic/Source/Source2010_ForumGeneva/KnowledgeBuilding2010_en.doc (7 November 2012)

Beacco, J.-C., Byram, M., Cavalli, M., Coste, D., Egli Cuenat, M., Goullier, F. & Panthier, J. (2010): Guide for the development and implementation of curricula for plurilingual and intercultural education. Retrieved from: http://www.coe.int/t/dg4/linguistic/Source/Source2010_ForumGeneva/EPI_Executive2010_EN.pdf (7 November 2012)

CDIP (Conférence suisse des directeurs cantonaux de l'instruction publique) (2011). Compétences fondamentales pour les mathématiques. Standards nationaux de formation. Adoptés par l'Assemblée plénière de la CDIP le 16 juin 2011. Retrieved from: http://edudoc.ch/record/96783/files/grundkomp_math_f.pdf (14 June 2011)

Council of Europe (2001): *Common European Framework of Reference for Languages: Learning, Teaching, Assessment*, 2001. Retrieved from: http://www.coe.int/t/dg4/linguistic/Source/Framework_en.pdf (7 November 2012)

DeSeCo (2005): The Definition and Selection of Key Competencies. Executive Summary. Retrieved from: http://www.deseco.admin.ch/bfs/deseco/en/index/02.parsys.43469.downloadList.2296.DownloadFile.tmp/2005.dskcexecutivesummary.en.pdf (7 November 2012)

KMK (2004): Bildungsstandards im Fach Mathematik für den Hauptschulabschluss nach Klasse 9, Retrieved from: http://www.kmk.org/dokumentation/veroeffentlichungen-beschluesse/bildung-schule/allgemeine-bildung.html#c7540 (7 November 2012)

OECD (2003). PISA 2003 Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills - Publications 2003. Retrieved from: http://www. oecd.org/dataoecd/46/14/33694881.pdf (7 November 2012).

OECD (2009). PISA 2009 Assessment Framework. Key competences in Reading, Mathematics and Science. Retrieved from: http://www.oecd.org/document/44/0,3746,en_2649_35845621_44455276_1_1_1_1,00.html (7 November 2012)

OECD 2010: PISA 2012 Mathematics Framework. Draft subject to possible revision after the field trial. Retrieved from: http://www.pisa.oecd.org/dataoecd/8/38/46961598.pdf (2 July 2011)

Pieper, I. (2011). Items for a description of linguistic competence in the language of schooling necessary for teaching/learning literature (at the end of compulsory education). An approach with reference points. Strasbourg: Council of Europe. Retrieved from: http://www.coe.int/t/dg4/linguistic/Source/Source2010_ForumGeneva/1_LIS-Literature2011_EN.pdf (7 November 2012)

Schweizerische Konferenz der Erziehungsdirektoren (EDK) (2011). Grundkompetenzen für die Mathematik. Nationale Bildungsstandards. Retrieved from: http://edudoc.ch/record/96784/files/grundkomp_math_d.pdf (7 November 2012)

Sinclair, J. M. H. & Coulthard, R. M. (1975): Towards an analysis of discourse: The English used by teachers and pupils. Oxford University Press, Oxford.

Thürmann, E., Vollmer, H. J. & Pieper, I. (2010): Languages of schooling: focussing on vulnerable learners. Council of Europe. Retrieved from: http://www.coe.int/t/dg4/linguistic/Source/Source2010_ForumGeneva/2-VulnerLLearnersThurm_EN.pdf (7 November 2012)

Vollmer, H. J. (2009). Language(s) in Other Subjects. Strasbourg: Council of Europe. Retrieved from: http://www.coe.int/t/dg4/linguistic/LangEduc/BoxD2-OtherSub_en.asp#LangOthSubE (7 November 2012)

Vollmer, H. J. (2010). Items for a description of linguistic competence in the language of schooling necessary for learning/teaching science (at the end of compulsory education). An approach with reference points. Strasbourg: Council of Europe. Retrieved from: http://www.coe.int/t/dg4/linguistic/Source/Source2010_ForumGeneva/1-LIS-sciences2010_EN.pdf (7 November 2012)

Vollmer, H. J. (2011). Schulsprachliche Kompetenzen. Zentrale Diskursfunktionen. Osnabrück: Universität Osnabrück. Retrieved from: http://www.anglistik.uni-osnabrueck.de/vollmer (7 November 2012)

Vollmer, H., Thürmann, E., Arnold, C., Hammann, M. & Ohm, U. (2008). Elements of a Framework for Describing the Language of Schooling in Subject-Specific Contexts: A German Perspective. Strasbourg: Council of Europe (Draft Version, 8.12.2008; to be published in 2012).

Winter, H. (1995). Mathematikunterricht und Allgemeinbildung. Mitteilungen der Gesellschaft für Didaktik der Mathematik, 61, 37-46.