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Mathematics and/as semiotic communication

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Disciplinarity versus discursivity? Mathematics and/as semiotic communication

Sigmund Ongstad

> (...) mathematics is used and can only be learned and taught as an integral component of a larger sense-making resource system which also includes natural language and visual representation. A semiotic perspective helps us understand how natural language, mathematics, and visual representations form a single unified system for meaning-making (Lemke, no date:1)

'Language' and 'reality'?

Human beings have on the one hand conquered the world by their capacity to develop and manipulate symbolic representations of their lifeworlds. On the other hand though this ability has become both a dependency and a blindness in that symbols have become 'real' and reality 'symbolic'. Following A. Schütz and his followers Berger&Luckmann as well as the so-called Sapir-Worf hypothesis the relationship between 'language' and 'reality' can thus be seen as reciprocal and hence paradoxical. These two notions are of course notoriously difficult to define. However one way to handle the interference of the two phenomena is (simultaneously) to conceive

a) language as a describable object
b) reality as a describable object
c) 'language' as a certain kind of reality and 'reality' as co-constructed by language

Starting from a) F. de Saussure can be seen as the key developer of the idea of verbal language as a closed linguistic system (la langue) as opposed to its use (la parole). Although different, the idea of a system, and language as a predisposition, was taken further in structural direction by N. Chomsky's generative approach. Formal and abstract grammaticalization of language thus supported the idea of language as an entity consisting of definable, recognizable and thus teachable sub-entities: This understanding is still rather dominating in traditional common sources for definitions. Language is (...) *a system of conventional spoken or written symbols by means of which human beings, as members of a social group and participants in its culture, communicate* (Britannica). *Language is a system of finite arbitrary symbols combined according to rules of grammar for the purpose of communication. Individual languages use sounds, gestures and other symbols to represent objects, concepts, emotions, ideas, and thoughts* (Wikipedia).

The very metaphor of 'language' tends to expand into other fields. Or, in some important cases 'natural' language is logically refined and redefined into new fields of knowledge:

In mathematics, logic and computer science, a formal language is a set of finite-length words (i.e. character strings) drawn from some finite alphabet, and the scientific theory that deals with these entities is known as formal language theory. Note that we can talk about formal language in many contexts (scientific, legal, linguistic and so on), meaning a mode of expression more careful and accurate, or more mannered than everyday speech (Wikipedia).

The structural tradition was paradigmatically opposed by a view that developed rather slowly from the late 1920s onwards, through the work of scholars such as Bühler, Jakobson, Mucharovsky, Firth, Bakhtin, Wittgenstein, Austin, Searle, Halliday and Habermas, all underpinning functional aspects of language. Language is language in use. Although in many ways different these theorists generally have argued that a pragmatic dimension is crucial and inevitable for understanding, not only what language might be, but how we learn 'it'. This position thus implies a functional view on how we relate to and perceive, not only what is called 'reality', but even to any disciplinary perception or further refining of it into new sciences and school subjects. Halliday thus showed how his own son Nigel as a toddler step by step was socialized *to* and *by* basic language functions. The academic awareness of these processes had the potential to change principally the perception of how 'language' and 'reality' interrelate, and it would even imply moving the principle scope from 'language' to 'communication'.

While most of these theorists tend to focus the pragmatic dimension as such, especially Bakhtin, Halliday and Habermas continued to work on the idea that the (micro) level of the communicative utterance or text needed to be related systemically, dialogically or reciprocally to some form of macro concept or phenomenon. While Bakhtin explored the role of *genre*, Halliday preferred to call his main macro phenomenon *register*. Habermas worked on a connection between communicational actions and *'lifeworlds'*. An important similarity between the three, regarding their view on how meaning is made, is that they insist on the basic role of *systemic* contexts, in other words that even *contexts* are structured by language (or rather, by 'societal' semiotics). In different other theories several contextual metaphors have been coined, such as *'umwelt'*, semiosphere, ecology, environment, fields, and systems. But these concepts have generally remained rather general. Even if they have been valuable for recognizing the necessity of a contextual understanding, they have not to any extent been differentiated further.

From the late 1960s onwards the systemic claim indirectly got support from a range of highly influential sociologists, such as Bernstein focusing *codes*, Giddens focusing *structuration*, Foucault focusing *discourse* and Bourdieu focusing *habitus*. By stressing this immanent macro level sociology thus helped moving the attention from the concrete action, utterance or text towards their cultural contexts - or rather - focusing *the interplay* between these two aspects or levels. Accordingly, in cultural studies and text theories 'reality' is no longer just what is directly focused by uttering or established sciences, but even the immanent communicative contexts accompanying the utterances.

Focusing on b) the relatively successful story of making linguistics into a scientific discipline rather than continuing being a philology, found its counterpart in a more conscious and determined will in the scientific closing of the mathematical sign. The closing helped natural science (and other new fields) in making dimensions of 'reality' into researchable, describable and understandable 'objects'. The precise semiotics of mathematical science thus created the foundation for and accordingly further generated the scientific revolution of the last centuries. However while science on the one hand has purified and strengthen its capacity to close (certain) phenomena and study them as objects, it is, in its strive to recruit and enculturate new generations of students and researchers on the other hand still dependent on the ways human beings learn and are socialized to this specific kind of knowledge.

Thus in the long historical 'journey' mathematics has got rid of 'natural' language. In particular mathematics has succeeded in becoming a pure meta-language by making it independent of impacts from the sender, the receiver and of context. The irony is that this independency has partly blinkered the historical connection between the disciplinarity and the discursivity of mathematics. For mathematics as an academic field this is a final victory, but for mathematics *education* it even represents a loss, since to enculturate children and novices to mathematics implies to bring them from the relative messiness of a culture's ethno-mathematics to the seemingly language-free disciplinarity of pure academic mathematics. That this teaching-learning enterprise has to be handled through the use of language, genres, discourses and semiotics, in short - communication, is a double irony.

This dilemma leads to point c). In the writings of among others Fluck (1992) and Vollmer (2006) there is a conscious play with the German words 'Fachlichkeit' and *'Sprachlichkeit'* hinting and theorizing close connections between the two. A related mutual relationship is conceptualized in the phrase "disciplinarily versus discursivity?". However, the question-mark expresses doubt about the value and validity of a direct polarization of the two phenomena. This is where the tradition after Schütz and others should be combined with the systemic, communicational, semiotic view already touched upon. According to a combined framework based on the empirical and/or theoretical work of these scholars, a broad, triadic concept of 'communication' can highlight and connect different aspects.

Language, communication or semiotics in (mathematics) curricula?

Traditional perceptions have for a long time dominated how the separated 'thing' called language has been handled in education outside the field of language. The-language-as-object-position even seems to dominate most current curricular thinking in Europe. 'Language' is a concept frequently used, but some times mis-conceptualized, while 'communication' is broader, although less used. 'Semiotics' is more appropriate, but less understood. Their intimate interrelatedness though is crucial, but in general often silenced. The following figure may help describing some basic relationships. This general framework aims at including *all* semiotics, not only verbal or textual communication.

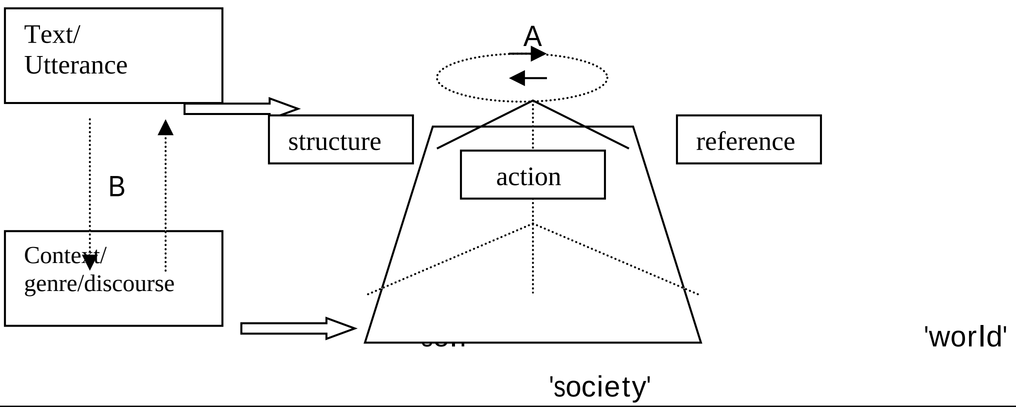

Figure 1. The principle relationship between the three major aspects on the concrete level of utterance/text (the top triangle) and their respectively corresponding three 'lifeworlds' aspects of the immanent level of context/genre, (the 'bottom part). The white arrows point to the two levels in the figure, the top surface and the rest. [For a more extended explanation, se Ongstad, 2006.]

Any utterance in any semiotic system consists of at least three aspects that is necessary to establish communication. A substantial form will have a certain syntactic structure. This structure functions as a signifier for a signified content and thus as a semantic reference. These two aspects connects to a third through its use, in which it becomes a pragmatic action or an act. The uttering self embodies the meaning potential through genres and discourses. Utterers have access to the some of the references to the 'world' and share some of these with others by which they form a society. All these aspects on both levels are working simultaneously, which means that this framework is consciously kept paradoxical - one has to differentiate between crucial parts, but one can not, because of a whole is not just a sum of parts.

Therefore when we *focus* on form we prioritize aesthetics. When we *focus* content, we prioritize epistemology and when we *focus* action we prioritize ethics although the other main aspects will always accompany any foregrounding ("the clarity and blindness of focusing"). Examples can be given from different national curricula: The Swedish curricula underline the importance of mathematics as aesthetics (Hudson and Nyström, 2007). If aesthetics is valued and prioritized, form, structure and syntax will be brought in the foreground. Further Singer (2007a) points to less weight in the new Romanian curricula to memorize and reproduce mathematical terminology (formal content elements). This represents a conscious shift within the semantic and epistemological aspects of the school subject.

Finally Pepin (2007a) shows how newer curricula in mathematics in the UK repeatedly underlines the importance of interpreting, discussing and synthesizing, almost on all course levels. The weight on such processes represents a strengthening of the pragmatic action aspects of language. In a general sense the question of which value acts and actions really have will, at the end of the day, be an ethical question. These three examples are of course general prototypes, any concrete utterance will be forced to be positioned somewhere in-between.

Hence we need to be equipped with an understanding of the relationship between language, communication and 'reality' that adequately can balance how meaning is structured, referred to and used in different specific contexts. A key question is how mathematics and mathematics education deal with this challenge.

What mathematics 'is' - and is not

Professions have played a key role in the development of disciplinarity - and vice versa. Within some disciplines the direct bindings to a profession or a field have over time been loosened and (re-)searching knowledge for its own sake has become a main driving force of a new, advanced kind of disciplinarity. For mathematics these historical shifts are symptomatic in the debates over the discipline's 'true nature'. While the relationship between science, technology and mathematics historically the last 200 years has been rather symbiotic, mathematics today serve so many different professions and fields, that a unified, valid definition of its 'nature' is hard to find.

Nevertheless, the history of the development of mathematics can, as already hinted, be portrayed as a slow process of liberation from the general ethno-mathematic culture, in which empirical practice and verbal communication played a constitutional role for how mathematics in the past was perceived and performed. The final break with empiricism and language generated a relatively independent discipline that defined and refined itself exactly by getting rid of the inadequate impreciseness of these two aspects.

A problematic consequence of defining mathematics in an essentialist way, is that the tacit relation to practice and language seems lost. In the descriptions of the many mathematical fields, as for instance found in Rusin (2004), there are hardly any explicit reference to language, semiotics or communication. This purification of the discipline to become forever context free is on the one hand the very reason for and a necessity for its success. On the other hand though it is perhaps one of the main obstacles for the *teaching and learning* of mathematics, a claim that will be developed more at length in the following.

Searching with Google for "definitions of mathematics" gives approximately 170.000 hits (July 2007). From a quite traditional and very general view mathematics is often seen as (...) *a science (or group of related sciences) dealing with the logic of quantity and shape and arrangement* (http://www.thefreedictionary.com/science). However such a characterisation only describes what, not how (or why). Hence methodological aspects that might be of significance, are not mentioned. A description that combines what and how (underlined in the quote by me) is found in Wikipedia where mathematics is seen as

(...) the body of knowledge centered on concepts such as quantity, structure, space, and change, and also the academic discipline that studies them. Benjamin Peirce called it "the science that draws necessary conclusions". Other practitioners of mathematics maintain that mathematics is the science of pattern, that mathematicians seek out patterns whether found in numbers, space, science, computers, imaginary abstractions, or elsewhere. Mathematicians explore such concepts, aiming to formulate new conjectures <u>and establish their truth by rigorous deduction from appropriately chosen axioms and definitions</u>
(http://en.wikipedia.org/wiki/Mathematics#Mathematics_and_physical_realityA; footnotes and links removed by SO)

When mathematics is understood in the broadest sense, not overstepping the thresholds to neighbouring academic disciplines, the field embraces between 60 and 70 different specific kinds or sub-branches of mathematics (of which for instance mathematics education is just one) (Rusin, 2004).

In a principle inquiry of definitions of mathematics Bonnie Gold identifies and discusses critically nine major claims (Gold, 2007). As a result of the inspection she outlines 13 criteria for 'good definitions'. Taken collectively these criteria seem to have a dual function, to describe (valid) internal cohesions within the discipline of mathematics and to relate what one could call mathematicallity to other disciplinarities. These two concerns are of course often closely related. Of the nine types of descriptions of mathematics there are hardly any that does not play some role in other disciplines. It is therefore not likely to find one *single* aspect that makes mathematics unique, and which can be used solely to define every former, present and future kind of mathematics.

As pointed to above a philosophical challenge for mathematics is that during its historical purification process, becoming an academic discipline, it tends to obliterate its own foundations. At the heart of the discipline as 'established' there seems to be a kind of safety-game where a 'universal givenness' of mathematics makes a critical questioning of the discipline irrelevant and inadequate. This intellectual 'laziness' (or this sensible pragmatic taken for granted attitude) is transmitted to mathematics education because mathematics of course here normally is based on and focuses the stability and not the slow development of the discipline. This tendency consolidates the idea that mathematics is given rather than developed and thus may function as another set of blinkers for how disciplinarity is generated.

Gold dismisses the claim that *mathematics is what mathematicians do*. Although she admits that one (...) *could modify it by saying that it is what mathematicians do when acting as mathematicians,* she doubts that one can avoid circularity when specifying what it is to act as a mathematician. However if one looks at this definition in the light of pragmatics (which Gold does not), it could be further refined. Mathematics as discipline could be described by the full set of practical and intellectual acts that are at work when doing mathematics (but not only). In other words, even mathematics needs to be seen, not just as products, but as processes. This will obviously accumulate into a long list, at least containing activities such as theorising, doing inductions and deductions, defining, arguing, calculating, giving premises, concluding, etc. This implies a *pragmatic* understanding of language and communication.

In discussions there at this point often tends to appear an opposition between applied and pure mathematics, where the kind of acts related to these types of doing mathematics are said to be qualitatively different (cf. paragraph C in Gold's paper, Gold, 2007). In any case the question of which mental and practical activities that are involved can not be finalised without a valid description of the content of mathematics (to the degree this is practically and principally possible). Gold finds that listing sub-fields is the most common way of defining mathematics.

Even if this gives some kind of concreteness to the question there are several dangers:

(...) such definitions risk becoming dated by the evolution of mathematics; even if we make our list include all the current Mathematics Reviews subject classifications, new subjects are being added all the time. Second, they emphasize the separateness of the different branches of mathematics, whereas if there has been any lesson from the development of mathematics in the last 50 years, it is the unity of mathematics, the complex web of interconnections between the supposedly different fields, even those which seem to have very different flavours (more on this in section IV). Third, they give no assistance in recognizing a new kind of mathematics when it appears (Gold, 2007).

In other words, from our perspective one should combine a *synchronic* and a *diachronic* view of the discipline, a conclusion which of course is close to the former that one needs to differentiate between *products* and *processes*. Nevertheless it takes into account the interplay between *stability* and *dynamics*.

Gold further claims that the difficulty with (...) *finding a common subject has caused people to turn to the methodology of mathematics to find its unifying theme, mathematics being unique among the sciences in making deductions from axioms the cornerstone of its reasoning* (Gold, 2007). The crucial role of axioms in mathematics is agreed upon in mathematics. Mathematics is built and continues to be built upon this particular genre. Metaphorically an axiom functions as a humming top in a supposedly eternal spin, so that it will never fall. From a (pragmatic) speech act perspective it can simplistically be described by an utterance beginning with *Given that...* It is the final preciseness, creativity and relevance of the description of the set of axioms that will bring mathematics further, closer to the cutting edge of its disciplinarity. But it is by the same token the continuous growth of (interrelated) axioms that makes mathematics stable. Paradoxically, using language to create a fixed point of departure is also what gives mathematics the imaginative freedom and makes 'pure' mathematics possible (and even free, fresh and fascinating). According to Gold Nevanlinna expresses a similar sentiment: *Mathematics combines two opposites, exactitude and freedom* (Nevanlinna, 1966:456).

Surprisingly, and for some, provokingly, this view makes language and mathematics to rather inherited (semiotic) phenomena. Hence while language in general and fiction i particular can be seen (with Umberto Eco) as the tool with which one in principle *can* lye, the regime of axioms in mathematics leads to the opposite, a position which is at the heart of Benjamin Peirce's famous definition of mathematics as the science which draws necessary conclusions.

In this perspective one of the foundations of mathematics is a purification of a particular kind of speech act where lying is made impossible. You can make mistakes, but not lye, once given the axioms that close the mathematical entities. Consequently, if you are lying or cheating deliberately, what you do is not (according to) mathematics. The main reason for that this is possible is the axiomatic closing of open signs. According to semiotic theory signs in 'natural' language are under the law of semiosis, an never-ending growth in the meaning of all concepts over time. In mathematics however such concepts/objects can not be part of an axiomatic act/definition.

One popular definition of mathematics is the discipline that studies 'patterns'. Gold (2007) argues that this view does not distinguish structures found in mathematics from other structures. Mathematics is for instance not interested in the patterns of atoms or molecules, rather, (...) *mathematics is concerned with the <u>properties</u> of patterns, the general relationships between patterns, how they behave, and so on.* To see mathematics as the science of patterns implies a structuralist perspective. Reuben Hersh, famous for advocating the (implicit pragmatic) view that mathematics is what mathematicians do, writes critically in *What Is Mathematics, Really?*:

The definition, “science of patterns,” is appealing. It’s closer to the mark than “the science that draws necessary conclusions” (Benjamin Peirce) or “the study of form and quantity”. (*Webster's Unabridged Dictionary*). Unlike formalism, structuralism allows mathematics a subject matter. Unlike Platonism, it doesn’t rely on a transcendental abstract reality. Structuralism grants mathematics unlimited generality and applicability. Structuralism is valid as a partial description of mathematics — an illuminating comment. As a complete description, it’s unsatisfactory (Hersh, 1997/Yiparaki, 1999:58).

Traces of language and communication in mathematics?

In the above searches for "what mathematics is" some few sources mention the concept 'foundation/s' as a main branch of the discipline (and not as something that is 'before' or 'outside' mathematics). In one of these different aspects of and threads to 'language' and 'semiotics' are symptomatically visible on and between the lines:

The term *foundations* is used to refer to the formulation and analysis of the language, axioms, and logical methods on which all of mathematics rests (see logic ; symbolic logic). The scope and complexity of modern mathematics requires a very fine analysis of the formal language in which meaningful mathematical statements may be formulated and perhaps be proved true or false. Most apparent mathematical contradictions have been shown to derive from an imprecise and inconsistent use of language. A basic task is to furnish a set of axioms effectively free of contradictions and at the same time rich enough to constitute a deductive source for all of modern mathematics. The modern axiom schemes proposed for this purpose are all couched within the theory of sets, originated by Georg Cantor, which now constitutes a universal mathematical language (Rusin, 2004, underlining by SO. The main point for underlining the above terms is to make aware some subtle and delicate relations between a discipline (here mathematics) and language.)

In the article The *Definition of Mathematics: Philosophical and Pedagogical Aspects* A. Khait defines language as a system of conventional spoken or written symbols by means of which human beings, as members of a social group and participants in its culture, communicate (Khait, 2005). After having discussed various perceptions of mathematics as they appear in literature, Khait suggests that (...) *mathematics is an essentially linguistic activity characterized by association of words with precise meanings* (Khait, 2005:137). It is not likely though that this position would get much general support among mathematicians.

In mathematics *education* though there seems to be a certain readiness for the question.

Niss (2003) describes the work of a Danish committee which through a project (KOM) attempted to answer the following question: *What does it mean to master mathematics*? To illustrate the endeavour, Niss offers an analogy, asking what is meant by 'literacy' or to master a language and use it in context. It includes to understand and interpret other people's oral speech, written texts produced by others and further to express oneself orally and in writing, all in a variety of different linguistic registers, and with reference to a variety of different forms and domains of oral and written 'texts'. Turning to mathematics Niss argues that:

To master mathematics means to posses mathematical competence. (...) *Mathematical competence* then means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy (Niss, 2003:6).

The project identified eight competencies which are said to form two groups. The first group concerns the ability to ask and answer questions in and with mathematics. The other is related to the ability to deal with and manage mathematical language and tools. In the following mainly the speech act verbs will be presented, but they are all connected to a more particular content parts of mathematics in schools and education. The point of focusing the verbs is to make aware the clear speech act character of these processes, and hence the functional, pragmatic aspects of the communication of his text.

In the first group one finds: Thinking mathematically: posing questions, understanding and handling the scope and limitations of a given concept, extending the scope of a concept, distinguishing between mathematical statements. Posing and solving mathematically: identifying, posing, specifying and solving different kinds of mathematical problems. Modelling mathematically: analysing foundations and properties of and decoding existing models, performing active modelling in a given context. Reasoning mathematically: following and assessing chains of arguments, knowing what a mathematical proof is (not), uncovering the basic ideas in a given line of argument, devising formal and informal mathematical arguments, and transforming heuristic arguments to valid proofs.

The second group consists of: Representing mathematical entities: understanding and utilising different sorts of mathematical representations, understanding and utilising relations, choosing and switching between representations. Making use of aids and tools: knowing the existence and properties of tools for mathematical activity, being able to reflectively use such aids and tools. Two of the competencies in this group deserve to be quoted at length:

(...) Handling mathematical symbols and formalisms such as
• decoding and interpreting symbolic and formal mathematical language, and understanding its relations to natural language;
• understanding the nature and rules of formal mathematical systems (both syntax and semantics);
• translating from natural language to formal/symbolic language

• handling and manipulating statements and expressions containing symbols and formulae.

(...) Communicating in, with, and about mathematics such as
• understanding others' written, visual or oral 'texts', in a variety of linguistic registers, about matters having a mathematical content;
• expressing oneself, at different levels of theoretical and technical precision, in oral, visual or written form, about such matters (Niss, 2003:8-9).

This presentation shows among others two things. Firstly that there actually exists an explicit understanding of the role of language and communication in mathematics (in the field of mathematics *education*). Secondly that the main perception is that even if language and communication are related, they are nevertheless separated. Separation is not only related to language and communication, but also to what is mathematics and what is not:

Furthermore, although the competencies are formulated in terms that may apply to other subjects as well, these terms are here to be understood in a strict mathematical sense. Thus we are talking about *mathematical* representations, not representations in general. Similarly, we are talking about *mathematical* reasoning, including proof and proving, not about reasoning in general like in general logic or in a court room, and we are talking about *mathematical* symbols, not other kinds of symbols such as icons or chemical symbols, let alone religious or literary symbols. In other words the competencies are specific to mathematics (Niss, 2003:9).

However, it is even clear that mathematics education needs to develop a more updated and adequate understanding of the relationship between discursivity and disciplinarity if (a new framework of) languages of education should be of any help to support teaching, learning and assessment in mathematics education.

Part of the problem of describing similarities between mathematics and language, and hence between disciplinarity and discursivity might be the way language is perceived (on both sides, so to speak). A core element in most disciplines is 'concept', the smallest unit of 'meaning' in a conceptual framework that normally will establish a field of knowledge and what Khait (2005) describes as *words with precise meanings*. In linguistics the smallest unit that carry meaning is the morpheme, in everyday layman discourse often simplified to 'word'. In semiotics however the point of departure is the sign.

Mathematics and mathematics education as Semiotics?

> (...) comprehensive view of curriculum is implicit in semiotics insofar as *all existing school subjects - and even subjects not yet formulated - are by their nature ways of organizing signs.* If we think of learners as individuals with the potential for understanding and communicating through a variety of signs (such as linguistic, gestural, pictorial, musical, and mathematical signs) and sign systems, we gain a fresh perspective both on human potential and on the organization of school
>
> (Suhor, 1991).

In his article *Mathematics in the Middle: measure, picture, gesture, sign, and word* Lemke argues that (...) *mathematics can best be learned and taught as an integral component of a larger sense-making resource system which also includes natural language and visual representation* (Lemke, no date:1). In his view formal and social semiotic perspectives can be used

(...) to show how natural language, mathematics, and visual representations form a single unified system for meaning-making in which mathematics extends the typological resources of natural language to enable it to connect to the more topological meanings made with visual representations (Lemke, no date:1).

After having discussed important aspects of the semiotics of mathematics Lemke concludes that the mathematics curriculum and education for mathematics teaching *(...) need to give students and teachers much greater insight into the historical contexts and intellectual development of mathematical meanings, as well as the intimate practical connections of mathematics with natural language and visual representation* (Lemke, no date:16)

The core unit in mathematics as a scientific discipline could accordingly be the (mathematical) sign. Mathematics is hence a semiotics in line with any other cultural artifacts. This semiotics has historically been established as a particular dynamic language system among a wider range of other different sign systems. This is achieved by the disciplinary use of the discursive and verbal genres axioms and definitions. These are used for the closing the imprecise meaning of signs, terms, notions, words and concepts used in everyday life. Accordingly mathematics is a meta language when seen as a system of 'given' entities, that is, defined, context-free, precise, interrelated terms and concepts. In this semantic sense scientific mathematics consists of a growing set of products. When considered syntactically the concepts form a particular interrelated disciplinary system. Perceived mainly as a pragmatic activity, doing mathematics implies semiotic processes in the sense that it uses and creates mathematical products, but even forms a methodology.

In mathematics education the use of mathematics is given a somewhat different function than in scientific, 'pure' mathematics devoted to the development of mathematics as a scientific discipline. However, when it comes to the process of solving problems there is no crucial difference between educational (or didaktic) use of mathematics and 'scientific' problem solving and the like.

Since the definition of the concept 'sign' is disputed, one needs to clarify how 'sign' relates to linguistic and everyday concepts such as 'word', 'discourse', and 'utterance'. Simplified one can differentiate between a closed and an open sign. For instance in 'natural' language the English 'and', the Norwegian *'og'* or the French 'et' could for example be seen as a relatively stable/partly closed sign or word caused by its high frequency and its basic function over time as a connector in everyday language. A mathematical reuse and hence a deliberate closing of this sign/word is the establishing of the mathematical sign ' + '.

Linguistics has described the core elements/units of spoken and written verbal language in terms of a system of phonemes, graphemes and morphemes. This will establish, in a semiotic sense, a basic sign level (although morphemes could/should be seen at a different level than phonemes/graphemes). On the 'next' level, both in mathematics and in 'natural' language, the signs become key elements in sentences and utterances. In both cases the created discourse establishes a semantic, epistemological proposition or the like. While closed, fixed, precise or defined signs and concepts are expected and inevitable in mathematics, the dynamic or open sign is the normal in 'natural' language. Thus on the next level the meanings of utterances in normal communication are dependent on context.

Introduction to mathematics education then, as initiation to mathematics as a school subject, is about bringing the novice, natural language users to understand the stability of mathematical signs and concepts. This happens by the accompanying use of didactic language in particular contexts from the dynamics and openness of 'natural' language and semiotics. At the other end of mathematics education, at the very cutting edge of research, mathematics is paradoxically about using the developed fixedness of the discipline to explore and to create new meanings, to solve (new) problems and to extend mathematics as discipline in new disciplinary contexts. On this long 'journey' three closely interrelated languages or semiotics interfere, the semiotics of mathematics as a discipline, the didactic communication as the language of the textbooks, curricula and teachers and the 'natural' language(s) of the learners.

The signs, words, the concepts, the terms, the vocabulary are the basic semantic aspects of both mathematics and language. How these elements can be combined can be described syntactically and what one can do or how these two semiotics can be used, is described pragmatically. A main challenge is that disciplinary descriptions of both mathematics and verbal language tend to leave out pragmatics. This may create problems. Firstly it is not really recognised that semantics, syntax and pragmatics are complementary. Secondly the pragmatics of both is often presented implicitly. Thirdly the role of dynamic contexts, that is, of specific disciplinary genres and discourses, tend not be recognized.

A semiotic, communicational view on teaching, learning and assessment also implies that a future framework for Languages of Education not only needs to address notions and concepts on two levels, utterance and genre (text and context or micro and macro) but even the intimate (dialogical) relationship between them. However it is not given that this work should start from scratch. The former CoE framework has opened a door for both a functional and a contextual understanding of language learning.

Language and communication in four national curricula

In the report from the mathematics group to COE (Ongstad, 2007b) it is claimed that mathematics probably will have more problems than most other school subjects with integration of mathematics and language and communication for several reasons. There is a strong will in the disciplinary parts of the curricula to describe mathematics rather than relating mathematics to different communicative contexts. Further, perceptions of language and communication seem rather fragmented and coincidental. Finally there are reasons to believe that among mathematics teachers there exists a strong sense of *disciplinarity* as purely mathematical. Mathematics is often conceived as a 'sky-scraper' rather than as a row of terraced houses, giving less room for seeing mathematic education as a compound of elements of aspects from other fields of knowledge.

Hence, mathematics as a discipline and mathematics education as a didaktic field has, at least in the four national curricula studied by the mathematics group, *not* yet really taken on the challenge of the intimate relationship between "Fachlichkeit und Sprachlichkeit". One of the reasons might be that language is generally objectified, rather than being seen as semiotic, relational and contextual. This means that an initiative to approach mathematics education with LAC needs to offer a presentation of the paradigmatic shifts concerning these relationships. This is nevertheless what is happening in many disciplines and fields of knowledge under notions such as the extended text concept, the discursive turn, the communicational shift etc.

The inspection of different definitions of mathematics as an academic field in this article makes it clear, at least in the view of most of the definers, that language, communication and semiotics play a minor role, if any, in how pure mathematics is perceived. This is not to say that discursivity is not important. The recognition should be highly relevant for introducing a new framework.

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